Some Notes on the Art of Theoretical Modeling in Economics
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Economics may be a science, but doing economics research is mostly an art. Science can be taught methodically, sequentially, deductively; skill in an art must be developed by studying examples, getting some suggestions, and most importantly, learning from one's own trial and error. In these brief notes I will try to offer some thoughts to guide your own efforts in theoretical modeling, using two of my favorite models from the research literature.

The art of scientific research differs from that of writing, painting, music etc. In the art of research, creativity cannot be free and unfettered; it is constrained by facts. You cannot have "surrealistic" science that violates reality. However, reality is complex, and its understanding is not always or even often best achieved by full detailed description. Art therefore enters the choice of what to include and what to leave out of one's analysis. It also enters the choice of techniques to use in the analysis. In theoretical modeling, which is my subject today, this can be various mathematical methods (algebra, geometry, calculus, etc.), and sometimes just plain words. One can explore and exploit analogies to other contexts. In empirical research, the methods are mainly statistical, and it is said that there are three important things: identification, identification, identification.

The art of theoretical modeling does need some technical background, in both economics and mathematics. It is essential that you have completed a good calculus-based intermediate micro and macro courses, liked them, and got good grades. It also helps if you have completed one or both of basic courses in Game Theory and the Economics of Uncertainty. Behavioral and psychological ideas are being increasingly used in economic theory; to familiarize yourself with them you would benefit from taking (concurrently) a course on the psychology of decision-making. Even with all this background, mastering the art of building your own theoretical models requires a lot of effort and experimentation.

If you have the interest, background, and determination to proceed, read on.

The first suggestion is obvious: begin by choosing a good question. Where do good questions come from? The best ones will usually come from some big puzzle that economic analysis has not yet successfully resolved. Some current event may suggest such a question; right now the interaction between finance and macroeconomics is one of the most important, interesting and active research areas. Growth and inequality, trade and employment, … there is no dearth of important economic questions for theoretical (and empirical) research. But there are tradeoffs to following hot trends; the risk or downside is that many people are working in these areas, and you may complete a good piece of work only to find that someone else has beaten you to the finish line. The research literature can be a good source of questions, but that may take you to third-order extensions of second-order extensions of extensions of work that was of limited interest to begin with.

When you have mapped out a general area of your interest, you can discover the literature from web searches; Google Scholar and Econ Lit are two of the best general sources; just type in various key words or phrases. And your adviser will suggest some literature or sources. Almost every paper you read will usually conclude with some suggestions for future research. Some of these may have been already explored by others; some may still be open. Find the paper on Google Scholar and click on the link “Cited by xxx” at the bottom left of that entry. This will show you all the papers that cite this one, and that will include ones that have followed up its line of research. Use these tools to identify the open questions that intrigue you. Then consult your adviser about their availability and feasibility. You may find some flaws or limitations in the previous work, and believe you can do better. Again take your ideas to your adviser.

Most questions are too complex to model in full detail. So after you have decided on your question, you have to map out a research strategy: Where to start? How to simplify? You have to develop some sense of what is essential to your concerns and what is incidental. This usually comes with some trial and error. So don’t get too attached to the first or even the second model you try to construct. Keep improving on it, changing it drastically, or even restarting from scratch. Of course you cannot do this for ever; you have to keep an eye firmly on the final deadline and work backward from it to figure out how much time you will need for various tasks of solving the model, interpreting the results, fixing any errors, and writing your paper or thesis chapter.

More suggestions are scattered in the specifics of the two models I will now discuss as exemplars of great theoretical research.

MARKET SIGNALING

This is the famous idea that won Michael Spence his Nobel prize in 2001. The starting point is Akerlof’s famous “lemons” model of market failure. You should have met this model in your intermediate micro courses; it is even mentioned in intro micro. The question is: Can markets find ways to solve the problem of information asymmetry, and with how much success?

The general idea is that mere assertions of quality are not credible; actions speak louder than words. But actions are (largely) governed by self-interest. An action can be credible proof of quality if taking that action would be in the interests of the owner of a good car but not in the interests of the owner of a bad car. An obvious example is offering a warranty; an owner who knows his car to be of good quality expects this to be less costly than an owner who knows his car to be a lemon. (But fulfillment of a warranty may be hard to enforce from a private owner.) Such an action that is initiated by the party to the transaction who has better information (here the owner selling the car) is called a “signal”.

The idea needs to be pursued further: What market equilibrium will result under signaling? Will it be efficient? …

The idea can also be explored in contexts other than the used car market. The one modeled by Spence is the job market. Workers know their own productivity (skills, work habits etc.) better than can a prospective employer. A worker who knows himself/herself to have high productivity wants to prove this fact credibly to the employer. This can be done to some extent by examination, but that is costly. So consider conceptual extreme case where direct testing is not feasible.

Can education serve as proof (credible signal) of high productivity? Education imparts some skills that raise productivity, but it can also prove existence of innate skills and also of attitude and capacity for hard work. For this to be a credible "separator," it should be the case that those who have the right skills etc. will find it optimal to achieve the required level of education and those without the right skills won't. Spence models this by assuming that the more skilled a person is, the less costly it is for him/her to acquire education, where the cost is an overall measure of many things: time spent and earnings forgone, mental burden of working hard, etc. What is more, the model isolates this aspect by ignoring all other aspects of education, e.g. genuinely raising productivity!

Here is the basic structure of Spence's model:

Group	Marginal product	Proportion of population	Cost C(y) of getting education y
1	1	q	y
2	2	1-q	y/2

Employers cannot know by direct observation or cost-effective examination which group any individual belongs to. They can observe his/her education level and have beliefs about group membership conditional on education. In equilibrium these beliefs must be confirmed by actual experience that results from hiring. Competition forces employers to pay each individual his/her expected marginal product. (So any monopsony/monopoly power in labor market is assumed away, to isolate the issue of signaling.)

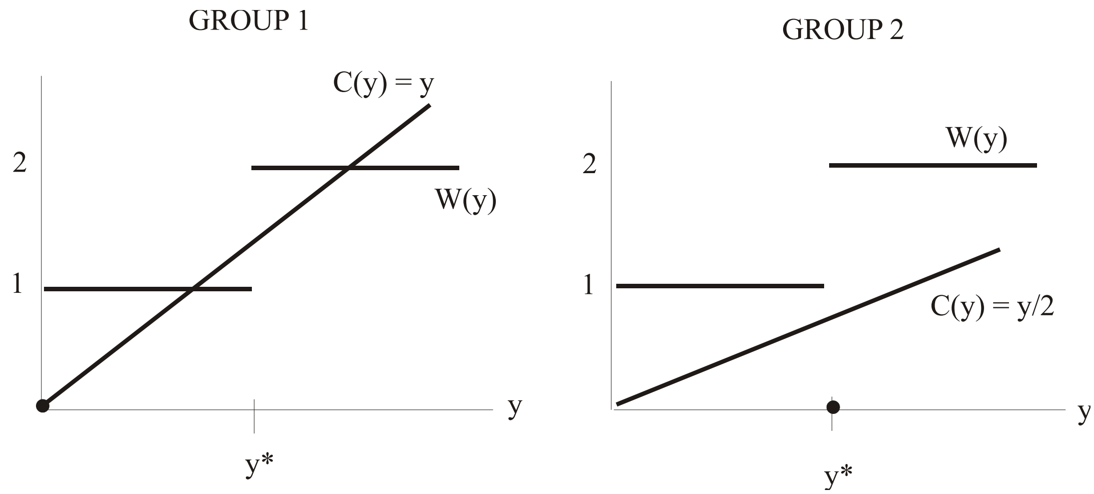
The discussion suggests an obvious form of belief to try out: There is a threshold y^* such that

Marginal product = 1 if $y < y^*$, and = 2 if $y \geq y^*$.

Given this belief, and the competition assumption, someone contemplating the education decision expects to face the following wage function

$W(y) = 1$ if $y < y^*$, and $= 2$ if $y \geq y^*$.

They choose y optimally to maximize $W(y) - C(y)$.



The optimal y is best found by inspection of the graphs of the functions W(y) and C(y). Don't use calculus: the functions are not differentiable. The only candidates (local maxima) are y* and 0. For the belief to be confirmed in equilibrium, we want Group 2 to choose $y \geq y^*$ and Group 1 to choose $y < y^*$. Therefore we need

$2 - y^*/2 > 1 - 0$, and $1 - 0 > 2 - y^*$, or $y^* > 1$ and $y^* < 2$.

Any y* in (1,2) can serve as a belief threshold to support an equilibrium that separates the two groups.

This gives us some results:

[1] There are multiple equilibria; those with $y^* > 1$ are plainly inefficient because a smaller y* would suffice. More "refined" equilibrium concepts suggest how the inefficient equilibria can be eliminated by employer experimentation.

[2] We have a more basic but unavoidable inefficiency: In the model, education has no intrinsic value. Group 2 incur the cost y*/2 solely to prove they are Group 2. This cost is a negative externality inflicted on them by the mere existence of Group 1! More generally, education will be taken beyond its optimal skill-increasing level because of its signaling value, but this is an unavoidable cost of the information asymmetry.

[3] There is yet another possible inefficiency: In the equilibrium, Group 1 get 1 and Group 2 get net payoff $2 - y^*/2$. Even in the best conceivable equilibrium ($y^* = 1$) this is 1.5. Consider the alternative where education is banned and everyone treated as average. With competition, employers will have to pay every worker the population-average marginal product,

$$q + 2\,(1-q) = 2 - q.$$

This is of course > 1 so Group 1 are better off in this regime, but so are Group 2 if $2 - q > 1.5$ or $q < 0.5$! If Group 1 is a sufficiently small proportion of the population, it is socially not worthwhile to pay the cost of separating Group 2. But that "pooling" can't be an equilibrium; individuals have the incentive to separate. Just a few of type 1 force the many of type 2 to incur a lot of cost of signaling!

This is just the first of many models in Spence's book (Market Signaling, Harvard University Press 1974) and article ("Job market signaling," Quarterly Journal of Economics 87(3), August 1973, 355-374). Read these for more details and extensions.

In my view the greatest merit of the model is that it not only answers initial motivating question (we see that markets can cope partially but not fully with information asymmetry), but also generates many rich ideas and results well beyond the initial question. Most importantly, it gives us the key insight: differential cost of the signaling activity is necessary to achieve separation. This proves useful in many other applications.

The model is very simple, stripped down to the barest essentials to isolate and clarify the working of signaling. (In fact Spence's thesis supervisor Thomas Schelling insisted that he simplify an initially more complex model which had a continuum of types, and repeatedly insisted on simplification "until the signaling goes away," to find out exactly what is essential for signaling to work.) But it leaves many possibilities for extension and generalization: continuum of types, multiple signals, multiple stages of the game with subsequent beliefs updated on the basis of earlier actions and in turn leading to choices of more actions …

BURDEN OF THE NATIONAL DEBT

The question that motivates this work is: Two conflicting views about the national debt were often expressed in media and political debates: [1] Debt will leave future generations poorer. [2] We owe it to ourselves. We pay taxes to service the debt, but this just turns into the return on the government bonds that we ourselves hold,[1] so the debt does not impose a burden on the economy as a whole. Which, if either, of these views is correct?

Any good economist will immediately recognize two starting points for analysis: [1] One must recognize that the "we" who pay the taxes may not be the same as the "we" who hold the bonds and get the returns. [2] The debt servicing obligation is more than a simple subtraction from the right hand side of the budget constraint. It changes relative prices including interest rates, and therefore affects marginal calculations and the equilibrium of the economy. The question is how to put all this into a model.

Diamond, in a famous paper (American Economic Review, 55(5), December 1965, 1126-1150), used and developed a framework now known as the "overlapping generations" model. Each person lives for two periods, works in the first (youth) and

[1] This argument used to be made by policy debaters on the political left. Now foreign governments and their citizen hold large quantities of US government debt. Diamond did include that possibility in his model; we will see a surprising implication of it.

lives on proceeds of savings in the second (old age). At any time, the population consists of one young and one old generation.

Note the stark and unrealistic assumptions. These are made solely to bring the central issues in sharp focus and obtain solutions with conceptual interpretations and insights; they can be (and have been) relaxed in later more complicated numerical models with many periods, uncertain lifetimes, etc. This is a good research strategy: make a simple start, figure out what is going on and why, and then gradually bring in more complex details of reality.

Number of young (workers) at time t is $L_t = L_0 (1+n)^t$. Let K_t denote the amount of productive capital at date t. Only K_0 is given; the subsequent path of accumulation must be found as a part of the solution. (General important point: Always keep clear track of what are the data or inputs or exogenous variables in the model, and what are the results or outcomes or endogenous variables.)

Output at date t is given by a production function $Y_t = F(K_t,L_t)$. This has constant returns to scale; therefore output per worker $y = Y/L$ can be expressed as a function of capital per worker $k = K/L$: $y_t = f(k_t)$ where $f(k) = F(k,1)$. The marginal product of capital is the interest rate: $r_t = F_K(K_t,L_t) = f'(k_t)$, and the wage is then

$$w_t = F_L(K_t,L_t) = f(k_t) - k_t f'(k_t).$$

(You may have already seen them in a basic growth theory class. If not, take them for granted for now.) Since K_0 and L_0 are data, w_0 and r_0 are known as well.

Consider a person born at time t. He/she works when young to earn wage w_t . Denote his/her consumption when young by c^Y_t . So the savings are $(w_t-c^Y_t)$. Next period when the person is old these earn a total return $(1+r_{t+1})(w_t-c^Y_t)$. Bequests are ignored, so this equals his/her consumption in period (t+1) when old by, denoted by c^O_{t+1} . So the intertemporal budget constraint is

$$c^O_{t+1} = (1+r_{t+1})\,(w_t-c^Y_t), \quad \text{or} \quad c^Y_t + c^O_{t+1} / (1+r_{t+1}) = w_t .$$

This just says that the discounted present value of consumption equals the (discounted present value of) wage income. Subject to this, the person chooses the consumption quantities to maximize a utility function $U(c^Y_t ,c^O_{t+1})$. That yields a saving function, say $s_t = H(w_t,r_{t+1})$. There are L_t workers like this. Their total savings must finance the capital next period, so $K_{t+1} = L_t\, s_t$. The demand for capital comes from entrepreneurs, who employ capital to the point where its marginal product equals the rate of interest. Therefore $r_{t+1} = f'(k_{t+1})$. Invert this to write the demand function for capital: $k_{t+1} = G(r_{t+1})$ say, or $K_{t+1} = L_{t+1}\, G(r_{t+1})$. Equating the supply and demand gives us the equilibrium condition

$$L_t\, H(w_t,r_{t+1}) = L_{t+1}\, G(r_{t+1}), \quad \text{or} \quad H(w_t,r_{t+1}) = (1+n)\, G(r_{t+1}). \qquad (*)$$

Substituting for w_t and r_{t+1} are respectively functions of k_t and k_{t+1} , this condition becomes a difference equation that governs the evolution of k_t starting from the given k_0.

This can be illustrated using Cobb-Douglas utility and production functions. If $U(c^Y{}_t, c^O{}_{t+1}) = \beta \ln(c^Y{}_t) + (1-\beta) \ln(c^O{}_{t+1})$, standard utility-maximizing calculations yield $c^Y{}_t = \beta w_t$, or $s_t = (1-\beta) w_t$. And if the production function is $f(k) = A k^\theta$, then

$$w = f(k) - k f'(k) = (1-\theta) A k^\theta ,$$

so the equilibrium condition is

$$(1-\beta) (1-\theta) A k_t^\theta = (1+n) k_{t+1} .$$

To solve this difference equation, take logs to linearize it

$$\ln(k_{t+1}) = \ln[(1-\beta) (1-\theta) / (1+n)] + \theta \ln(k_t).$$

Because $\theta < 1$, the dynamics converges to a "steady state of growth" with a constant capital-labor ratio k* given by

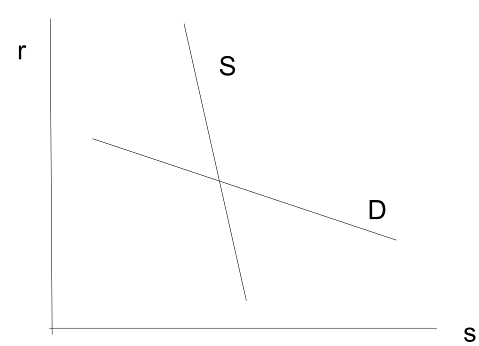
$$(1-\theta) \ln(k^*) = \ln[(1-\beta) (1-\theta) / (1+n)] .$$

All the other magnitudes of economic interest can then be solved in terms of the k's.

With more general production and utility functions, we will need to investigate stability of the dynamics and impose a convergence condition; Diamond's paper does this. Here I show only the "normal" case. I take the steady state of (*):

$$H(w,r) = (1+n) G(r) \qquad (**)$$

and use $w = f(k) - k f'(k)$ and $f'(k) = r$ to express w in terms of r (the "factor price frontier". Then in the figure below the curve labeled S is the supply of saving (left hand side of (**), the curve labeled D is the demand for saving (the right hand side).



Before introducing national debt into the model, let us ask if the equilibrium is Pareto optimal. For this, we compare it to the choice of a benevolent planner who is concerned to maximize the consumers' utility. The planner can split output Y at time t into consumption for the two types and capital accumulation:

$$Y_t = C^Y_t + C^O_t + (K_{t+1}-K_t) ,$$

or in per worker terms,

$$L_t\, y_t = L_t\, c^Y_t + L_{t-1}\, c^O_t + (L_{t+1}\, k_{t+1} - L_t\, k_t) .$$

(Remember that the old at time t were born at time (t-1) so there are only L_{t-1} of them.) Dividing through by L_t , we have

$$y_t = c^Y_t + c^O_t /(1+n) + (1+n)\, k_{t+1} - k_t .$$

To keep the calculation simpler I will consider this only in a steady state, although that is a bit deceptive. (The economy does not have the choice of steady states as such, because it is constrained by its initial capital endowment. But you can learn the full dynamic story later.) In a steady state with a constant capital-labor ratio k, we can omit the time subscripts to write the equation as

$$c^Y + c^O /(1+n) = y - n\, k .$$

Subject to this, the planner wants to maximize $U(c^Y ,c^O)$. Since $y = f(k)$, the k should be chosen to maximize the right hand side, so $f'(k) = n$. Then the marginal rate of substitution between consumption when young and when old should equal (1+n). Thus, both from the consumption side and the production side, the rate of interest should equal the rate of growth. This is called the "golden rule of accumulation" and the resulting steady state the "golden rule" state.

There is simply no reason why the equilibrium we calculated above should coincide with this. In the market equilibrium steady state, the rate of interest r is given in terms of the parameters β, θ, and n, and will equal n only by coincidence. In fact, if the market interest rate is < n, the steady state will be clearly Pareto inefficient: there is too much capital and everyone can benefit by decumulating some of it. This inefficiency has to do with the fact that the economy has no definite and finite time horizon. If it did, then the usual result on the Pareto efficiency of competitive equilibrium would hold. Further work has been done to explore the relation between such inefficiency and asset bubbles, and conditions under which such bubbles can be ruled out.

Now introduce government debt into this model. Denote the debt at time t by B_t , issued as short-term (one period) bonds, and the debt per worker by $b_t = B_t/L_t$. Diamond uses a setting in which taxes are levied on wage income, and taxes X_t at time t go toward paying the interest on the debt at time (t+1). Therefore $B_{t+1} = B_t + r_t\, B_t - X_t$.

I will do the analysis (and for most part so does Diamond) in the context of a steady state, with a constant ratio b of debt per worker. Then the equation expressed in per worker terms becomes $(1+n)b = b + rb - x$, or $x = (r-n)b$. Then the wage net of taxes is $w-(r-n)b$, and saving per worker is now $s = H(w-(r-n)b,r)$.

If the debt is external (owed to foreigners), the saving must be used for holding capital as before, and the equilibrium condition is

$$H(w-(r-n)b,r) = (1+n)\,k = (1+n)\,G(r)$$

in the same notation for these functions as used in the corresponding condition (**) in the absence of debt. Remember that w and r are functions of k, so this equation defines the steady state k for given exogenous n and b. If $r > n$, then $w-(r-n)b < w$, so for any given k the saving (left hand side) is less than that in the absence of debt (so long as the marginal propensity to save out of net wage income is positive, or consumption when old is a normal good). In the Figure, the S curve shifts to the left. The steady state equilibrium interest rate r is higher, and the steady state equilibrium k is smaller than it would be in the absence of debt. Therefore the debt not only reduces disposable income directly, but also has the secondary harmful effect of reducing capital and thus output. So we see the point made earlier about the importance of working out the full equilibrium implications of the existence of debt. Of course the debt may be used for financing some public capital like infrastructure that shifts the whole production function and thereby has a beneficial effect, but it can also be used for financing “bridges to nowhere”.

If the debt is internal (“owed to ourselves”), the saving must finance both the physical capital and the bonds: $L_t\, s_t = K_{t+1} + B_{t+1}$. Therefore the steady state equilibrium condition is

$$H(w-(r-n)b,r) = (1+n)\,(k+b) = (1+n)\,[G(r)+b]\ .$$

The right hand side is larger than in the case of external debt. So in the Figure the D curve shifts to the right, further raising the equilibrium interest rate r and further lowering the equilibrium capital/labor ratio k. Paradoxically, debt owed to ourselves has an even more harmful secondary effect of reducing capital accumulation! Room must be made in the domestic residents’ portfolios for the financial assets in addition to the physical capital. This is partly achieved by higher equilibrium interest rate, and therefore less steady state physical capital.

Diamond gives a nice compact summary of his results: “External debt has two effects in the long run, both arising from the taxes needed to finance the interest payments. The taxes directly reduce available lifetime consumption of the individual taxpayer. Further, by reducing his disposable income, taxes reduce his savings and thus the capital stock. Internal debt has both of these effects as well as a further reduction in the capital stock arising from the substitution of government debt for physical capital in individual portfolios.”

The somewhat surprising result is another nice feature of the model. It seems counterintuitive at first, because we think that debt owed to our own citizens should be less damaging to the nation's economy than debt owed to foreigners. But once we think through the result, the intuition that financial assets crowd out physical assets in the citizens' portfolios makes perfectly good sense. It is often this way with intuition. It must be built up and modified by rigorous analysis and hard thinking about results of formal models.

As with the signaling model, this model can be, and has been, modified, extended, and applied to the point that it has become a "workhorse" framework for modern macroeconomics. You will meet it (if you haven't already) throughout modern macro and growth courses and find uses for it in your own research.

SUMMING UP

The two models illustrated some of the art of building theoretical models, and served as vehicles for making some general points. One I would like to emphasize in conclusion is the virtue of simplicity. Each model tries to capture the essential aspect of the question being posed – how does signaling work, and how is the equilibrium of saving and capital accumulation with different generations affected by the existence of debt – in the simplest possible context – two types of individuals for signaling with no other productive aspects of the signal, and a two-period overlapping generation setting.

Remember what Hannibal Lecter told Clarice Starling in *The Silence of the Lambs*: "Read Marcus Aurelius. The emperor advises simplicity. Of each particular thing ask: What is it in itself? What is its nature?" And in more modern idiom, the famous economist Hal Varian (of UC Berkeley and Google) in his article "How to Build an Economic Model in Your Spare Time," gives the memorable acronymic advice "KISS: Keep It Simple, Stupid." (http://people.ischool.berkeley.edu/~hal/Papers/how.pdf)

This applies to modeling that seeks to obtain basic understanding of some issue and to develop an intuition for it. The next step, or the focus of your project, may be to take an existing model where the basic conceptual matters are already understood, and make it more realistic to the point of obtaining numerical solutions using computer simulation. For that purpose, simplicity is not necessarily a virtue.

Incidentally, Diamond says in his essay in Szenberg and Ramrattan (2014) that his model grew out of teaching the national debt issue in his public finance course. So research and teaching can be valuable complements, not substitutes.

FURTHER READING:

Memoirs and advice articles from several eminent economists:

Michael Szenberg (editor) Eminent Economists: Their Life Philosophies. Cambridge University Press, 1992.

Michael Szenberg (editor) Passion and Craft: How Economists Work. University of Michigan Press, 1998.

Michael Szenberg and Lall B. Ramrattan (editors) Eminent Economists II: Their Life and Work Philosophies. Cambridge University Press, 2014.

A small idiosyncratic sample of other influential and instructive theoretical modeling papers:

David Baron and Roger Myerson, “Regulating a monopolist with unknown cost,” Econometrica 50(4), July 1982, 911-930.

Vincent Crawford and Joel Sobel, “Strategic information transmission,” Econometrica 50(6), November 1982, 1431-1451.

Avinash Dixit, “Entry and exit decisions under uncertainty,” Journal of Political Economy 97(3), June 1989, 620-638.

Rudiger Dornbusch, “Expectations and exchange rate dynamics,” Journal of Political Economy 84(6), December 1976, 1161-1176.

Gene Grossman and Elhanan Helpman, “Protection for sale,” American Economic Review 84(4), September 1994, 833-850.

Michael Kremer, “The O-ring theory of economic development,” Quarterly Journal of Economics, 108(3), August 1993, 551-575.

Robert Lucas and Edward Prescott, “Equilibrium search and unemployment,” Journal of Economic Theory 7(2), February 1974, 188-209.