ECO 317 – Fall 2009
Economics of Uncertainty

# Midterm Examination

**Important instructions:** [1] This exam is open-print, but closed-electronic. You can bring any books, notes, etc. printed on paper, but should keep all electronic equipment off: computers, calculators, cell phones, ... Any numerical calculations needed should be carried out using only paper and pencil (or pen). [2] For your convenience, the point total is 80, so you should plan to spend approximately one minute per point. [3] Show the steps of your logic and math clearly. Answers that appear by magic, or jumping over steps, will not get credit. [4] **Print** your name clearly on the front cover of each answer booklet. Write your answers clearly. We won't grade unreadable answers. [5] Show the steps of your calculations and arguments. [6] Write and sign the honor pledge – "I pledge my honor that I have not violated the honor code during this examination." – on the front cover of your answer booklet.

**There are four questions on three pages.**

## Question 1 — 25 points

NOTE: In this question your drawing of the probability triangle does not have to be accurate to scale, but should show the absolute and relative positions of the points with reasonable clarity.

Consider a risky prospect with three sure consequences $C_1 = 1$, $C_2 = 2$, and $C_3 = 4$, measured in millions of dollars. Their respective probabilities $p_1$, $p_2$, and $p_3$ are variables.

(a)In the right-angled probability triangle diagram with $p_1$ on the horizontal axis and $p_3$ on the vertical axis, show a contour of equal expected monetary value. What is its slope?

(b) In the same diagram, show the points corresponding to the following four lotteries:

| Case | $p_1$ | $p_2$ | $p_3$ |
|---|---|---|---|
| A | 0 | 1/2 | 1/2 |
| B | 1/4 | 0 | 3/4 |
| C | 1/2 | 0 | 1/2 |
| D | 0 | 1 | 0 |

(c) An individual ranks these cases as follows: A is indifferent to B, B is preferred to C, and C is indifferent to D. Answer the following questions. Do not derive any general results that support your answer, but state them clearly.

(i) Do these preferences show risk-averse behavior? Why, or why not?

(ii) Are these preferences consistent with expected utility? Why, or why not?

(iii) Is D second-order stochastic dominant over C? Why, or why not?

## Question 2 — 20 points

On the eve of the football game against Yale, your initial wealth is $W_0 = 1000$. The chance of Princeton winning is 50:50. You can bet at fair odds, taking either side to win, and betting any amount of money you choose. You are an expected utility maximizer, and your utility of consequences depends on your final wealth $W$ and on whether Princeton wins (P) or Yale wins (Y). Specifically,

$$u(W, P) = \ln(500 + W), \qquad u(W, Y) = \ln(W)\,.$$

(a) Show that at any level of initial wealth and without any bets, you prefer Princeton to win.

(b) Find the optimal amount $X$ that you will bet on Princeton to win (so if $X$ turns out to be negative, you are betting against Princeton).

(c) Having made the optimal bet, do you still prefer Princeton to win?

(d) Briefly state the intuition for the result in (c).

## Question 3 — 15 points

You have an initial wealth of \$1,000, and face a 25 percent probability of losing \$500 of it. You can by insurance at a given load factor $\lambda$, choosing the amount of indemnity optimally. Your utility-of-consequences function is $U(W) = \ln(W)$. You are asked to find how high $\lambda$ must be if you choose not to buy any insurance. You are advised to use the following steps (a)-(d), but can use a different method if you prefer.

(a) Write down the expression for your final wealths $W_1$ and $W_2$ in the no-loss and loss state, and for your expected utility $EU$, as functions of the amount of indemnity $X$, involving the parameter $\lambda$.

(b) Write down the expression for the derivative $EU'(X)$.

(c) If the optimum is at $X = 0$, what does that imply about $EU'(0)$

(d) In turn, what does that imply about $\lambda$?

## Question 4 — 20 points

You work for a boss who cannot observe your effort directly. However, your effort affects the boss's profit, and your compensation depends in part on the profit. Specifically, if you make effort $X$, the boss's profit will be

$$Y = X + \epsilon\,,$$

where $\epsilon$ is normally distributed with mean 0 and variance $v$. Your compensation is given by

$$C = w + s\,Y\,,$$

where $w$ is a fixed wage and $s$ is your profit-share. Your objective is to maximize a mean-variance utility function with quadratic disutility of effort:

$$u = \mathbf{E}[C] - \tfrac{1}{2}\,a\;\mathbf{V}[C] - \tfrac{1}{2}\,k\,X^2\,,$$

where $\mathbf{E}[C]$ denotes the expectation and $\mathbf{V}[C]$ denotes the variance of $C$.

(a) Find the formulas for $\mathbf{E}[C]$ and $\mathbf{V}[C]$ as functions of $X$ (also involving some or all of parameters $w$, $s$, $v$, $a$, $k$).

(b) Find the formula for $u$ as a function of $X$ (and parameters).

(c) Hence find a formula for your optimal choice of $X$ as a function of parameters.

(d) Find a formula for $u$ as a function of $w$, $s$ and the other parameters $v$, $a$, $k$ when $X$ is chosen optimally as in (c).