ECO 317 – Fall 2007
Economics of Uncertainty

Midterm Examination – Answer Key

The distribution of total points was as follows (remember these are out of a max of 80, not 100):

70-79	60-69	50-59	40-49	< 40
9	6	1	3	1

The question-by-question performance summary was as follows:

Statistic	Q. 1	Q. 2	Q. 3	Q.4	Aggregate
Minimum	5	10	4	5	27
Maximum	20	19	20	20	79
Average	15.05	15.65	15.6	17.4	63.7
Median	16	16	17	20	69

Question 1 — 20 points

Common errors: In part (c) (i), some hypothesized that since D is indifferent to C and A is indifferent to B while having smaller risk and lower return, the person is risk-averse. It wasn't explained e.g. why A has smaller risk than B. The answer that was expected required noticing the fact that the slopes of indifferent curves is higher than that of equal monetary payoffs; see the sample midterm Q.1(b). In (ii), some didn't point out or prove that indifferent curves are parallel. Most difficulty, however, was caused by last question, part (iii), where people forgot to check if means are the same. This was a big issue in Problem Set 3 Q.2.

(a) (4 points) The contours of constant expected monetary values are

$$1 \, * \, p_1 + 2 \, * \, (1 - p_1 - p_3) + 4 \, * \, p_3 = \text{constant}$$

or

$$2 \, * \, p_3 - p_1 = \text{constant}$$

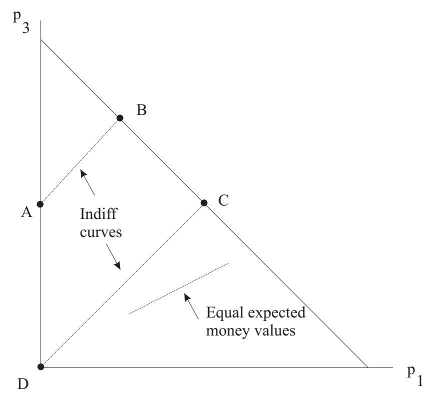
so in the (p_1, p_3) diagram these have slope = 1/2. See figure below (2 points for this part of the figure):

(b) (4 points) The four points are shown in the same figure.

(c) The lines AB and CD are parallel straight lines. A is indifferent to B and C to D. So from observing these four points only, indifference curves can be parallel straight lines.

(i) (4 points) The indifference lines (slope = 1) are steeper than the lines of equal expected monetary value (slope = 1/2), so the behavior is compatible with risk-aversion.

(ii) (4 points The indifference lines are parallel, so the behavior is consistent with expected utility. (Note: Further observations may yield evidence that contradicts this, for example A



may be indifferent to some point E not on the line AB. But that is a separate issue; here we are only asked the check the compatibility of the four points given.)

(iii) (4 points) C is a spreading out of the "probability distribution" in D, but the spread is not mean-preserving: the mean of the distribution of outcomes in C is 2.5 whereas that in D is obviously 2. Therefore D is not SOSD over C.

Question 2 — 20 points

Common errors: (1) Not mentioning the second-order condition. (Just 2 points) (2) Intuition did not mention marginal utilities. (This was discussed with the corresponding question in the precept.)

(a) (2 points) For any W,

$$u(W, P) = \ln(500 + W) > \ln(W) = u(W, Y)\,.$$

Therefore at any level of initial wealth and without any bets, you prefer Princeton to win.

(b) (10 points) If you bet X on Princeton to win at fair odds, your expected utility is

$$EU(X) = \tfrac{1}{2}\ \ln(500 + 1000 + X) + \tfrac{1}{2}\ \ln(1000 - X)\,.$$

To maximize this with respect to X, the first-order condition is

$$EU'(X) = \tfrac{1}{2}\ \frac{1}{1500 + X} + \tfrac{1}{2}\ \frac{-1}{1000 - X} = 0\,.$$

$EU(X)$ is concave so the SOC is met. Solving the FOC,

$$1000 - X = 1500 + X, \qquad \text{or} \qquad X = -250\,.$$

you bet \$250 against Princeton.

(c) (4 points) Having made the optimal bet, your final wealth is \$750 if Princeton wins and \$1250 if Princeton loses, and your utilities of consequences are

$$u(750, P) = \ln(500 + 750) = \ln(1250) = u(1250, Y)\,.$$

So you are indifferent whether Princeton wins or loses.

(d) (4 points) Without bets, even though your total utility is higher when Princeton wins, your marginal utility is smaller:

$$u'(W, P) = \frac{1}{500 + W} < \frac{1}{W} = u'(Y, P) .$$

That is why you bet against Princeton. And with this specific functional form, you are able to "hedge perfectly": if Princeton loses, you get just enough money to compensate you for your utility loss.

Bonus credit 2 points for those who remember the sample midterm and make the following comparison: In the sample exam, for any given level of wealth not just your total utility but your marginal utility when Princeton wins is twice as much as that when Yale wins. Therefore there you bet on Princeton, and that further increases your utility difference in the two cases.

Question 3 — 20 points

Common errors: A few people still couldn't set up the expected utility correctly. There were some numerical mistakes. More important, several people did not express the first-order condition as the correct inequality, using instead either equality or strict inequality.

(a) (6 points) If you buy X of indemnity, your premium is $P = 0.25\,(1+\lambda)\,X$, so your final wealth in the no-loss state is

$$W_1 = 1000 - 0.25\,(1+\lambda)\,X ,$$

and that in the loss state is

$$W_2 = 1000 - 0.25\,(1+\lambda)\,X - 500 + X = 500 + [1 - 0.25\,(1+\lambda)]\;X .$$

Your expected utility is therefore

$$EU(X) = 0.75\;\ln\{1000 - 0.25\,(1+\lambda)\,X\,\} + 0.25\;\ln\{500 + [1 - 0.25\,(1+\lambda)]\;X\,\} .$$

(b) (4 points) The derivative is

$$EU'(X) = 0.75\;\frac{-0.25\,(1+\lambda)}{1000 - 0.25\,(1+\lambda)\,X} + 0.25\;\frac{1 - 0.25\,(1+\lambda)}{500 + [1 - 0.25\,(1+\lambda)]\;X} .$$

(c) (4 points) For $X = 0$ to be optimum, the condition is

$$EU'(0) \leq 0.$$

(d) (6 points) In turn, this implies

$$0.75\;\frac{-0.25\,(1+\lambda)}{1000} + 0.25\;\frac{1 - 0.25\,(1+\lambda)}{500} \leq 0 .$$

Multiplying through by 1000,

$$-\tfrac{3}{16}\ (1+\lambda) + \tfrac{1}{4}\ 2 - \tfrac{2}{16}\ (1+\lambda)] \leq 0\,,$$

or

$$\tfrac{1}{2} \leq \tfrac{5}{16}\ (1+\lambda)\,,$$

or

$$1+\lambda \geq \tfrac{16}{10} = 1.6, \qquad \text{or} \qquad \lambda \geq 0.6\,.$$

Question 4 — 20 points

By the way, remember this example; we will use it when we do the theory of incentives later in the term.

(a) (6 points) $C = w + s\,(X+\epsilon) = (w + s\,X) + s\,\epsilon$. Therefore

$$\mathbf{E}[C] = w + s\,X\,,$$

and

$$\mathbf{V}[C] = s^2\,v\,.$$

(b) (2 points) Substituting,,

$$u = (w + s\,X) - \tfrac{1}{2}\,a\,s^2\,v - \tfrac{1}{2}\,k\,X^2\,.$$

(c) (5 points) To maximize u with respect to X,

$$du/dX = s - k\,X = 0\,,$$

so $X = s/k$.

(d) (7 points) Substituting, the resulting u (indirect utility function) is

$$\begin{aligned} u &= w + \frac{s^2}{k} - \tfrac{1}{2}\,a\,s^2\,v - \tfrac{1}{2}\,k\ \frac{s^2}{k^2} \\ &= w + \tfrac{1}{2}\ \frac{s^2}{k} - \tfrac{1}{2}\,a\,v\,s^2 \end{aligned}$$