ECO 317 – Fall Term 2009
Economics of Uncertainty
Sample Final Examination

Important instructions:

[1] This exam is open-print, but closed-electronic. You can bring any books, notes, etc. printed on paper, but should keep all electronic equipment off: computers, calculators, cell phones, ... Any numerical calculations needed should be carried out using only paper and pencil or (preferably) pen.

[2] The exam has **four** pages; make sure you have them all.

[3] The exam is in two sections, A and B. Write your answers to questions in these sections in separate, clearly marked, booklets. Within each section you can write your answers to the questions in any order, but keep all the pages on which you answer any one question together. Failure to follow these instructions may lead to loss of points in grading.

[4] For your convenience, the point total is 180, so you should plan to spend approximately one minute per point. At the end of the allotted three-hour time, you can "buy" extra time at the rate of 4 points per minute.

[5] **Print** your name clearly on the front cover of each answer booklet. Write your answers clearly. We won't grade unreadable answers.

[6] Show the steps of your calculations and arguments. Use well-labeled diagrams where appropriate, and explain any symbols or notation you introduce.

[7] Write and sign the honor pledge – "I pledge my honor that I have not violated the honor code during this examination." – on the front cover of your Section A answer booklet.

SECTION A

Question 1: (30 points)

(a) What is the Ellsberg Paradox? Is behavior that exhibits this paradox consistent with expected utility theory? Why, or why not?

(b) Briefly outline Rabin's calibration theorem. What are its implications for expected utility theory?

Question 2: (25 points)

Consider a risk-averse expected utility maximizer with a twice-differentiable utility-of-consequences function $V(W)$, where W denotes his random final wealth. Define his coefficient of absolute risk aversion.

Suppose such a person is risk-averse and has a given non-random initial wealth W_0. He can invest in a riskless asset with a sure total return R_0 per dollar invested, or in a risky

asset with a random total return R per dollar invested. The random variable R has a given density function $f(R)$ over a given support $[0, \overline{R}]$. The investor chooses the amount X to be invested in the risky asset. Formulate his optimization problem. Assume that the optimum is interior ($0 < X < W_0$). Find the first order condition, and verify that the second order condition is satisfied.

If the investor has constant absolute risk aversion, show that his optimal X is independent of his initial wealth W_0.

SECTION B

Question 3: (30 points)

There are two equally likely states of the world. In state 1, your endowment of wealth will be 12; in state 2, it will be 3. You are an expected utility maximizer with a utility-of-consequences function $V(W) = \sqrt{W}$ where W denotes your final wealth. You can buy and sell state-contingent claims to wealth at prices 1/3 for claim to one unit of state-1 wealth and 2/3 for claim to a unit of state-2 wealth.

(a) Write down the expression for your expected utility.

(b) Write down the budget constraint for your choices of final wealth quantities W_1 and W_2 in the two states.

(c) Find your optimal choices of W_1 and W_2. (You need not check the second-order conditions.)

(d) Repeat the exercise with a logarithmic utility-of-consequences function. Give an intuitive explanation for the difference in the results of the two cases.

Question 4: (30 points)

You have hired a contractor to do a job using your materials. However, the contractor can, by expending effort, reduce the amount of materials used. There are two types of contractors, labeled $i = 1, 2$. Type i has private information about his efficiency parameter k_i, and the labels are chosen so that $k_1 < k_2$. The probability that the contractor is type i is p_i , where each of the p_i is positive and $p_1 + p_2 = 1$. If type i expends effort e_i , the cost of materials used will be $c_i = k_i - e_i$. The contractor's effort is not verifiable. The fact of completion of the job, and the cost of materials used, are observable and verifiable. The type k_i of the contractor is his private information, so you cannot infer the effort e_i from observation of the cost c_i.

The utility function of a contractor of either type is $u = t - e$, where t is the payment he receives from you and e is the effort he expends. The outside opportunity of a contractor of either type is 0.

(a) What kind of information asymmetry is involved in this situation? Moral hazard, or adverse selection, or both?

You offer a menu of two contracts, where you intend type i to select contract i.

(b) What is the form of the contracts, compatible with the information limitations?

(c) What are the incentive compatibility constraints on your contract choices?

(d) What are the participation constraints on your contract choices?

You want to minimize your expected expense consisting of the payment to the contractor and the cost of materials,

$$p_1\ (\,t_1 + c_1\,) + p_2\ (\,t_2 + c_2\,)\,.$$

(e) Among the contracts satisfying the incentive compatibility and participation constraints for both types, show that your expected expense is the lowest when $t_1 + c_1 = t_2 + c_2 = k_2$.

(f) Contrast this with the hypothetical first-best case where you know the type of the contractor. Compare your expected cost and the two types utilities in the two cases.

Question 5: (30 points)

Note: When doing the calculations in this question, work with fractions. Do not try to convert them to decimals by hand. Remember that you are not allowed to use calculators.

Consider Spence's job market signaling model with the following specifications. There are two types of workers, 1 and 2. The productivities of the two types, as functions of the level of education E, are

$$W_1(E) = E\,, \qquad W_2(E) = \tfrac{3}{2}\ E\,,$$

The costs of education for the two types, as functions of the level of education, are

$$C_1(E) = \tfrac{1}{2}\ E^2\,, \qquad C_2(E) = \tfrac{1}{3}\ E^2\,.$$

Each worker's utility equals his/her income minus the cost of education. Companies that seek to hire these workers are perfectly competitive in the labor market.

(a) If types are public information (observable and verifiable), find expressions for the levels of education, incomes, and utilities of the two types of workers.

Now suppose each worker's type is his/her private information.

(b) Verify that if the contracts of part (a) are attempted in this situation of information asymmetry, then type 2 does not want to take up the contract intended for type 1, but type 1 does want to take up the contract intended for type 2, so "natural" separation cannot prevail.

(c) Leaving the contract for type 1 as in part (a), what is the range of contracts (education-wage pairs) for type 2 that can achieve separation? You are given the following information:

$$(\,3 - \sqrt{5}\,)/2 = 0.382 \quad \text{and} \quad (\,3 + \sqrt{5}\,)/2 = 2.618\,.$$

(d) Of the possible separating contracts, which one do you expect to prevail? Give a verbal but not formal explanation for your answer.

Question 6: (35 points)

Choose any situation within the health care system that involves moral hazard. For your choice, discuss briefly (in less than 500 words), (a) how well or poorly the US system copes with the problem, (b) how well or poorly other systems in other countries cope with the same problem, (c) what solution you would recommend and why. Your arguments should be grounded in the relevant economic theory. You should not propose utopian solutions that ignore information asymmetries or assume human perfectibility. Moreover, you should recognize that the solution you propose for the chosen problem may interact with and possibly aggravate other problems with the system.

Your answer will be graded taking into account: [1] the correctness and cogency of your facts and arguments, [2] the use you make of economic theory in your reasoning, [3] the overall organization of your essay. Many of these elements are subjective, and our decision in these matters is final.
