Important instructions: [1] This exam is open-print, but closed-electronic. You can bring any books, notes, etc. printed on paper, but should keep all electronic equipment off: computers, calculators, cell phones, ... Any numerical calculations needed should be carried out using only paper and pencil (or pen). [2] For your convenience, the point total is 80, so you should plan to spend approximately one minute per point. [3] Show the steps of your logic and math clearly. Answers that appear by magic, or jumping over steps, will not get credit. [4] Print your name clearly on the front cover of each answer booklet. Write your answers clearly. We won’t grade unreadable answers. [5] Show the steps of your calculations and arguments. [6] Write and sign the honor pledge – “I pledge my honor that I have not violated the honor code during this examination.” – on the front cover of your answer booklet.

There are four questions on two pages.

Question 1 — 20 points

Consider a risky prospect with three sure consequences $C_1$, $C_2$, and $C_3$, listed in the increasing order of your preferences. Regard these as fixed, and the respective probabilities $p_1$, $p_2$, and $p_3$ as variables. In the right-angled probability triangle diagram with $p_1$ on the horizontal axis and $p_3$ on the vertical axis, show how you can depict the following concepts. In each case, draw a rough sketch showing the lotteries and/or indifference curves that illustrate the concept, and give a brief verbal and/or mathematical explanation of it.

(a) Independence Axiom.
(b) Risk aversion for an expected utility maximizer
(c) First order stochastic dominance.

Question 2 — 20 points

On the eve of the football game against Yale, your initial wealth is $W_0 = 1000$. The chance of Princeton winning is 50:50. You can bet at fair odds, taking either side to win, and betting any amount of money you choose. You are an expected utility maximizer, and your utility of consequences depends on your final wealth $W$ and on whether Princeton wins (P) or Yale wins (Y). Specifically, $u(W, P) = 2W^{1/2}$, $u(W, Y) = W^{1/2}$.

(a) Show that at any level of initial wealth and without any bets, you prefer Princeton to win.
(b) Find the optimal amount $X$ that you will bet on Princeton to win (so if $X$ turns out to be negative, you are betting against Princeton).
(c) Having made the optimal bet, do you still prefer Princeton to win?
**Question 3 — 15 points**

You are an expected utility maximizer with a logarithmic utility-of-consequences function. Your initial wealth is $W_0$. You can invest $X$ of this in an asset that will return to you either double the amount you invested or half the amount you invested, with equal probabilities. The rest, $W_0 - X$ stays as cash with certainty. Find the algebraic expression your optimal choice of $X$ as a function of $W_0$.

Show that the elasticity of your optimal choice of $X$ with respect to your initial wealth $W_0$ equals 1. Give an intuitive reason for this result.

**Question 4 — 25 points**

Cara Bell owns a small farm. She has to decide the quantity $Q$ of wheat to plant before the price $P$ of wheat is known. The distribution of $P$ is normal with mean $\mu$ and standard deviation $\sigma$. Her cost of growing $Q$ is $\frac{1}{2} h Q^2$, where $h$ is a positive constant. So her income (profit) $Y$, expressed as a function of the choice variable $Q$ and the random variable $P$, is

$$Y = P Q - \frac{1}{2} h Q^2$$

Her utility-of-consequences function is

$$V(Y) = - \exp(-a Y)$$

where $a$ is a positive constant (and has the interpretation as her coefficient of absolute risk aversion). She chooses $Q$ to maximize expected utility.

Show that her optimal choice is given by

$$Q = \frac{\mu}{h + a \sigma^2}$$

Note: You should use without proof the result that if $P$ is normally distributed with mean $\mu$ and standard deviation $\sigma$, and $k$ is a constant, then

$$E[ \exp(k P) ] = \exp( k \mu + \frac{1}{2} k^2 \sigma^2 )$$