ECO 317 – Economics of Uncertainty – Fall Term 2009
Problem Set 1 – Due October 1

Question 1: (10 points)

Three fair dice are rolled. A gambler argues as follows. There are six different ways in which the number of spots can sum to 9, namely

$$1,2,6; \quad 1,3,5; \quad 1,4,4; \quad 2,3,4; \quad 2,2,5; \quad 3,3,3.$$

There are also six ways in which the number of spots can sum to 10, namely

$$1,3,6; \quad 1,4,5; \quad 2,2,6; \quad 2,3,5; \quad 2,4,4; \quad 3,3,4.$$

Therefore the two events "the spots sum to 9" and "the spots sum to 10" should have equal probabilities.

Is his reasoning correct? If not, what are the actual probabilities?

Question 2: (10 points)

Another gambler argues as follows. "When a fair die is rolled once, the chances of getting six are 1/6. Therefore if the same die is rolled four times, the chances of getting six at least once are $4/6 = 2/3$." In terms of probability theory, precisely what is his error? What is the correct probability?

Is the event "getting six at least once in four throws of one die" more or less likely than "getting at least one double-six in 25 throws of a pair of dice"? What about 26 throws of the pair of dice?

Question 3: (10 points)

An urn contains six balls, three white and three blue. Each ball has a number painted on it. Two of the white balls and two of the blue balls carry the number 1, and one of each color carries the number 2. One ball is drawn at random from the urn. Are color and number independent?

Question 4: (20 points)

(a) A highly contentious issue is before the U.S. Senate. Forty-five of the senators are committed to one side and forty-five to the other. The remaining 10 will decide independently randomly, and for each, there is equal probability $\frac{1}{2}$ that he/she will vote on the one side or the other. What is the probability that the Vice-President will have to cast a tie-breaking vote?

(b) What if the number of committed senators are 40 on each side, and the remaining 20 will act in the way that the 10 did in part (a)?

Question 5: (30 points)

A random variable X follows a normal distribution with mean μ and variance σ^2.

(a) Write down its probability density function.

(b) Calculate, showing the steps of the calculation, the expected value of e^{kX} where k is a given constant.

Question 6: (20 points)

Suppose you have a computer program that can generate random numbers that have a uniform distribution on $[0, 1]$. You want to generate random numbers that follow a different probability law, with a known cumulative distribution F that is strictly increasing on support $[a, b]$. Outline a procedure for doing so.

Hint: Let $X : S \mapsto R$ be a random variable with a uniform distribution on $[0, 1]$, so its cumulative distribution function say H is given by $H(t) = t$ for $t \in [0, 1]$. Consider another random variable Y, defined on the same sample space S as is X, by

$$Y(s) = F^{-1}(X(s)),$$

and consider the cumulative distribution function of Y, say G.