ECO 317 – Economics of Uncertainty – Fall Term 2009
Problem Set 2 – Due October 15

Question 1: (30 points)

Consider a situation of uncertainty with three possible outcomes, namely money rewards of amounts 1, 2 and 3. (Think of these as millions of dollars if you like.) The probabilities are denoted by p_1, p_2 and p_3 respectively. The rewards are going to be kept fixed, and the probabilities are going to be variables. (5 points for each part below.)

(a) In the probability triangle diagram with p_1 on the horizontal axis and p_3 on the vertical axis, sketch a curve along which the expected monetary reward is a constant. What is the slope of this curve?

(b) Mickey is an expected utility maximizer, and he attaches utilities $u(1) = 1$, $u(2) = 4$, and $u(3) = 6$ to the monetary rewards. Which of the two alternatives, outcome 2 for sure, and a 50:50 gamble between outcomes 1 and 3, does Mickey prefer?

(c) In the same probability triangle diagram, sketch an indifference curve for Mickey. What is the slope of this curve?

(d) Leo is also an expected utility maximizer, but he attaches different utilities $u(1) = 9$, $u(2) = 12$, and $u(3) = 18$ to the three consequences. Which of the two alternatives, outcome 2 for sure, and a 50:50 gamble between outcomes 1 and 3, does Leo prefer?

(e) In the same probability triangle diagram, sketch an indifference curve for Leo. What is the slope of this curve?

(f) Here you are asked to infer a general principle from your findings in (a)-(e) above. Instead of the numbers for consequences and utilities, use general symbols $c_1 < c_2 < c_3$ for the monetary reward amounts, and $u(c_1)$, $u(c_2)$, and $u(c_3)$ for their utilities. Find the equations for contours of constant expected monetary reward and of constant expected utility. Find a condition involving the c_i and the $u(c_i)$ under which the latter contours are steeper. Express this condition in a way that tells you something about this person's attitude toward risk.

Question 2: (10 points)

Consider the utility-of-consequences function

$$u(c) = 1 - \exp(-c^2/2)\,.$$

Find the expression for its coefficient of absolute risk aversion, $-u''(c)/u'(c)$. Is the person with this utility function risk-averse?

Question 3: (20 points)

Define the *coefficient of absolute risk tolerance* as $-u'(c)/u''(c)$, the reciprocal of the coefficient of absolute risk aversion. One would think that risk tolerance increases as the initial position gets better, that is, the coefficient is an increasing function of c. Suppose it increases linearly:

$$-\,u'(c)/u''(c) = a + b\,c$$

where a and b are constants, $a \geq 0$ and $b > 0$. Find the class of all utility-of-consequences functions u that have this property.

Question 4: (40 points)

You are enrolled in the course Optimal Dynamic Decisions 101. ODD 101 has weekly (12) problem sets, but allows you to drop any one without penalty. To work on a problem set, you have to give up some other activity. You do not know in advance how appealing the other activity will be in future weeks. You only know that the utility from each week's alternative activity is uniformly distributed over $[0, 1]$, and the utilities in different weeks are independent. Each week in which you receive a problem set, you also find out the utility you will get from that week's alternative activity, and then decide whether to drop that problem set. You want to choose your dropping strategy to maximize the expected utility of the alternative activity. (There is no time-discounting across weeks during the term.)

(a) (2 points) Suppose you have not yet used up your freedom to drop a problem set, and n problem sets are to come. Denote by $V(n)$ your expected utility from outside activity in this situation, before you receive that weeks's problem set and learn the actual utility of that week's alternative activity. Show that when the utility of the alternative activity that week, $u \in [0, 1]$, is revealed to you, you will use your drop opportunity if and only if u exceeds the threshold $u^*(n) = V(n-1)$.

(b) (9 points) Derive the recursive equation linking successive $V(n)$s:

$$V(n) = \tfrac{1}{2}\,[\,1 + V(n-1)^2\,]\,.$$

Hint: When evaluating the expectation, you must carefully separate the range of integration $[0, 1]$ into two parts corresponding to your different optimal choices.

(c) (5 points) The initial condition is $V(0) = 0$. Using Excel or some other method, solve the difference equation in (b). So tabulate $V(n)$ and $u^*(n)$ as n ranges from 1 to 12. Comment on your results.

(d) (2 points) Suppose a kind-hearted professor takes over ODD 101 and increases the number of problem sets you can drop from 1 to 2. Now you can maximize your expected sum of utilities from the drop opportunities (again, without any time-discounting over the weeks). Denote by $V(m, n)$ this expected total utility when you have m drop opportunities in hand and n weeks remain. Show that your decision is to drop a problem set if that week's utility is revealed to exceed the threshold $u^*(m, n) = V(m, n-1) - V(m-1, n-1)$.

(e) (12 points) Derive the equation linking successive $V(m, n)$s:

$$V(m, n) = V(m-1, n-1) + \tfrac{1}{2}\,\{\,1 + [V(m, n-1) - V(m-1, n-1)]^2\,\}\,.$$

What are the initial conditions (immediately obvious values of $V(m, n)$ for small m and n)?

(f) (5 points) Using Excel or any other method, tabulate $V(m, n)$ and $u^*(m, n)$ for $m =$ 1, 2, $n =$ 1, 2, … 12. Comment on your results and their comparison with those in Part (c) above.

(g) (5 points) What features of reality were ignored in the above model. Briefly (without doing any math) suggest how they would affect the results.