ECO 317 – Economics of Uncertainty – Fall Term 2009
Problem Set 3 – Due October 22

**Question 1:(15 points)**

Prove that if a distribution function $F_1(W)$ is first-order stochastic dominant over another distribution $F_2(W)$, then $F_1$ yields a higher mean of $W$ than does $F_2$.

Produce an example to show that the converse is not true.

**Question 2: (25 points)**

Consider a lottery with three scalar outcomes $C_1 = 1$, $C_2 = 2$, and $C_3 = 3$ (think of these as alternative wealth amounts $W$ measured in millions of dollars), with respective probabilities denoted by $p_1$, $p_2$, and $p_3$. Consider a particular probability distribution, call it $A$, with $p_1 = 0.3$, $p_2 = 0.5$, $p_3 = 0.2$. You are asked to compare it to another probability distribution $B$ that has the same three outcomes, but general algebraic values $(p_1, p_2, p_3)$ of the probabilities.

(a) Regarding $W$ as a continuous variable that can range from 0 to 4, draw a rough sketch of the cumulative distribution function $F(W)$ of $A$. (Your sketch here, as well as those in parts (c) and (d) below, do not have to be very accurate to scale, but should clearly convey the basic requirements.)

(b) Show the point corresponding to A in the probability triangle diagram.

(c) Show the set of points (it will constitute an area) in this diagram where $B$ must be if $B = (p_1, p_2, p_3)$ is to be first-order stochastic dominant over A? (Hint: The distribution function you sketched in part (a) will be useful in thinking about when $B$ can be FOSD over A.)

(d) Show the set of points (this time it will be a straight line) in the same diagram where $B$ must be if $B = (p_1, p_2, p_3)$ is to be second-order stochastic dominant over A? (Hint: The procedure you followed in part (c) should suggest what you need to do here.)

**Question 3: (30 points)**

Consider an expected utility maximizing risk-averse individual with the utility-of-consequences function $u(W)$ and initial wealth $W_0$. There is a lottery that pays $G$ with probability $p$ and $B$ with probability $(1 - p)$, with $G > B$.

(a) Suppose the individual already owns the ticket to this lottery, in addition to his initial wealth. Find an equation that implicitly defines the smallest price $P_s$ such that he would be willing to sell the ticket for this price.

(b) Now suppose he does not initially own the ticket, and has to consider whether to buy one. Find an equation that implicitly defines the highest price $P_b$ that he would be willing to pay to buy the ticket.

(c) Calculate $P_b$ and $P_s$ if the individual's utility-of-consequences function of his final wealth $W$ is $V(W) = \sqrt{W}$, and $G = 10$, $B = 0$, $W_0 = 10$, $p = 0.5$. (This requires a good calculator.)

(d) Prove that if the individual's utility-of-consequences function $V(W)$ has constant absolute risk aversion, then $P_b = P_s$.

**Question 4: (30 points)**

This question is about the comparative static effect of a change in initial wealth on the allocation between a riskless asset and a risky asset. We did an almost identical calculation and proof in the class, involving the absolute amount of the allocation to the risky asset, and obtained a condition involving the coefficient of absolute risk aversion. Here you have to retrace the same steps, but the choice variable is the *fraction* of initial wealth invested in the risky asset and the comparative statics depends on the behavior of the coefficient of *relative* risk aversion. Also note that in the class we assumed the absolute risk aversion coefficient to be a decreasing function of wealth; here you are told to assume that the relative risk aversion coefficient is an increasing function of wealth, with the corresponding difference in the result about the portfolio choice.

Consider an expected utility maximizer who has a concave utility-of-consequences function $u(W)$ defined over his final wealth. Here concavity is interpreted to mean $u''(W) < 0$ for all $W$. He has initial wealth $W_0$, and can invest a fraction $Y$ of this in a risky asset yielding a random total rate of return (1 plus the interest/dividend/capital gain or loss) $R$ with a density function $f(R)$ defined over the domain $[R_L, R_H]$. The rest, $W_0\,(1 - Y)$, is invested in a safe asset yielding a non-random total rate of return $R_0$.

(a) Find the expression for his expected utility.

(b) Find the first-order condition for $Y$ to maximize expected utility.

(c) Is the second-order condition satisfied?

(d) Assume that the maximum is a regular interior one (with $0 < Y < 1$) defined by the first-order condition. Suppose $W_0$ increases slightly. Prove that if the coefficient of relative risk aversion $r(W) = -W\; u''(W)\,/\,u'(W)$ is an increasing function of $W$, then the increase in $W_0$ leads to a decrease in the optimal $Y$.