

ECO 317 – Economics of Uncertainty – Fall Term 2009  
Problem Set 4 – Due November 19

**Question 1: (30 points)**

Do Exercise 10.1, p. 167 from Eeckhoudt-Gollier-Schlesinger.

**Question 2: (35 points)**

Consider two college roommates, Doc the premed, and Geek the computer science major. They recognize that a doctor will have a steady income, whereas a computer scientist's income will depend on the fate of his dotcom. They have both just completed ECO 317, so they propose to trade Arrow-Debreu securities to achieve an optimal allocation of the risk. Assume that each acts as a price-taker in the markets for Arrow-Debreu securities.

They have calculated that Doc's wealth will be \$10 million no matter what. Geek's wealth will be \$50 million if his dotcom flourishes, and 0 if it collapses to a mere  $\cdot$ . The probability that the dotcom flourishes happens is 40%, and the probability that it collapses is 60%.

(a) (2 points) What are the "states of the world," or "scenarios" for short? What are the aggregate wealth amounts in the various scenarios?

(b) (4 points) An Arrow-Debreu security for each scenario is defined as a contract that will pay 1 megabuck in that scenario and nothing otherwise. What are the two students' endowments of Arrow-Debreu securities? Writing  $P_i$  for the price (in today's trading) of the Arrow-Debreu security for the scenario labeled  $i$ , write down the values of the two students' endowments of these securities.

(c) (4 points) Writing  $W_i^j$  for the final wealth of the student labeled  $j$  (where  $j$  stands for Doc or Geek) in the scenario labeled  $i$  (that is, he ends up with  $W_i^j$  after the scenario is realized and the claims implied by his trades in Arrow-Debreu securities have been settled), what are the two students' budget constraints for today's trades in Arrow-Debreu securities?

(d) (4 points) Each has a logarithmic utility-of-consequences function of wealth. Write down expressions for the two students' expected utilities.

(e) (12 points) Maximize the expected utility of each subject to his budget constraint, to find their demand functions for these securities.

(f) (9 points) Find the equilibrium relative price of the securities, and the magnitudes of the two students' final wealths in the scenarios.

(Note: This question assumes that the two students act as price-takers in the markets for the two Arrow-Debreu securities. Since there are only two of them, you will doubt the validity of this assumption. Just accept it for now; I will comment on it in the answer key.)

**Question 3: (35 points)**

Markets are being held today, and uncertainty will be resolved in a year's time. There are three possible states of the world, labeled Animal, Vegetable, and Mineral, and three assets consisting of shares of stock in three companies, Hunters, Gatherers, and Diggers. The total

dollar returns that will be realized in a year's time in different states of the world from holding one share of each of the three stocks are given by the following table:

Assets	States of the world		
	Animal	Vegetable	Mineral
Hunters	100	75	60
Gatherers	80	125	60
Diggers	100	50	150

The shares are traded in perfectly competitive markets today, before you know which state of the world will come about. Suppose the equilibrium price in today's markets of one share of Hunters stock is \$70, that of one share of Gatherers stock is \$75, and that of one share of Diggers stock is \$110.

(a) (10 points) Arrow-Debreu securities for each of the three states of the world are traded in perfectly competitive markets today. Let  $P_a$ ,  $P_v$ , and  $P_m$  denote their respective prices. Express the equilibrium price of each of the three companies' shares in terms of the equilibrium prices of the Arrow-Debreu securities. Hence find the equilibrium prices of the Arrow-Debreu securities.

(b) (7 points) Find the sure rate of interest between now and next year.

(c) (10 points) Find the linear combination of  $X_H$  shares of Hunters,  $X_G$  shares of Gatherers, and  $X_D$  shares of Diggers that will replicate one Arrow-Debreu security for the Animal state of the world. Use this as an alternative way of finding the equilibrium price of that security, and verify that you get the same answer as you did in part (a).

(d) (8 points) Define a new asset, "a call option on the Diggers stock with a strike price of 90." This is defined as a contract that gives you the right, but not the obligation, to buy one share of Diggers for \$90 in a year's time, after the uncertainty gets resolved and you know which of the three states of the world has come about. Find the price of this new asset in today's market.