ECO 317 – Economics of Uncertainty – Fall Term 2009
Problem Set 6 – Answer Key

The distribution was as follows:

90-99	80-89	< 80
11	2	2

Only one general comment: Review the concept of the single crossing property - what it is and what it accomplishes in mechanism design problems. This comment also applies to Problem Set 7.

Question 1: (70 points)

Part (a) Moral hazard:

(i) (6 points) Comparing the expected utilities net of effort cost in the two situations, the customer will make good effort if

$$.25 \ \ln(W_B) + 0.75 \ \ln(W_G) - k \geq .5 \ \ln(W_B) + .5 \ \ln(W_G)$$

or

$$.25 \, [\, \ln(W_G) - \ln(W_B) \,] \geq k$$

or

$$\ln(W_G) - \ln(W_B) \geq 4\,k, \qquad \text{or} \qquad W_G \geq W_B \ e^{4k}$$

(ii) (2 points) The insurance company's zero expected profit line when the customer makes bad effort is

$$0.5 \ W_B + 0.5 \ W_G = 0.5 \ (W_0 - L) + 0.5 \ W_0 = 0.5 * 1 + 0.5 * 4 = 2.5, \qquad \text{or} \qquad W_B + W_G = 5$$

(iii) (2 points) The equation for the insurance company's zero expected profit line when the customer makes good effort is

$$0.25 \ W_B + 0.75 \ W_G = 0.25 * 1 + 0.75 * 4 = 3.25, \qquad \text{or} \qquad W_B + 3 \ W_G = 13$$

(iv) (5 points) At the local optimum where the customer is making bad effort, there is full insurance at the fair price corresponding to probability 0.5 of loss (from the general results of the analysis done in class). So $W_B = W_G$ along the zero expected profit line in (ii). That is, $W_B = W_G = 2.5$, and then the customer's utility is $\ln(2.5) = 0.916$.

(v) (8 points) At the local optimum where the customer is making good effort, the zero profit condition in (iii) applies and the incentive condition for good effort derived in part (i) holds as an equality (again from the general results of the analysis done in class). Solving those equations jointly for W_B and W_G, we have

$$W_B = \frac{13}{1 + 3\,e^{4k}}, \qquad W_G = \frac{13 \ e^{4k}}{1 + 3\,e^{4k}}$$

The utility can be evaluated from either of the two expressions for expected utility in (i) since the two are equal at this point. The second is simpler, and yield

$$0.5 \ln \left[\frac{13}{1+3\,e^{4k}} \right] + 0.5 \ \ln \left[\frac{13\ e^{4k}}{1+3\,e^{4k}} \right] = \ln(13) - \ln(1+3\,e^{4k}) + 2\,k$$

(vi) (8 points) Comparing the final expressions for utility at the local optima in (v) and (iv), we see that the local optimum in (v) is globally optimum if

$$\ln(13) - \ln(1+3\,e^{4k}) + 2\,k \geq \ln(2.5)$$

or

$$5.2\ e^{2k} \geq 1 + 3\,e^{4k}$$

By substituting $Z = e^{2k}$ and solving the resulting quadratic in Z, or graphing the two functions, or whatever other method you choose, this yields $k \leq 0.207$ approximately.

(vii) (9 points, 3 for each case - no insurance, the optimal insurance contract, and the hypothetical ideal) Now $k = 0.1$. The customer's expected utility where he gets no insurance at all is

$$0.25\ \ln(1) + 0.75\ \ln(4) - 0.1 = 0.9397$$

and the sure wealth that will yield this utility is $\exp(0.9397) = 2.559$. (Note that the no-insurance point is located in the region where it is optimal for the customer to exert good effort. So we know that with no insurance, the customer is going to exert good effort; so we need to subtract the cost of effort from the utility.)

Expected utility in the the globally optimal contract is given by the formula in (vi)

$$\ln(13) - \ln(1+3\,e^{4*0.1}) + 2*0.1 = 1.06467$$

and the sure wealth that will yield the same utility is $\exp(1.06467) = 2.8998$.

In the hypothetical ideal optimum where effort is observable, and the customer is given full insurance at the statistically fair price conditional on his making good effort, the customer gets full insurance along the zero expected profit line of (iii), so $W_B = W_G = 3.25$, and expected utility is $\ln(3.25) - 0.1 = 1.07865$. The sure wealth that will yield the customer the same utility is $\exp(1.07865) = 2.941$.

Additional information; you were not asked to derive this. With all this calculation, we can quantify the social cost of the information asymmetry. It is best done in "equivalent wealth" units because that is tangible whereas the utility scale is a theoretical construct. Comparing the optimal contract to no insurance, the customer gains $2.8998 - 2.559 = 0.3408$ units of equivalent wealth. If the information asymmetry did not exist, he would have received full insurance and made good effort, gaining $2.941 - 2.559 = 0.382$ units of wealth. Thus, because of the information asymmetry, the customer is able to achieve only $0.3408/0.382 = 0.891 = 89.1\%$ of the ideally conceivable gains. So the social cost of the information asymmetry equals 10.9% of the ideally conceivable gain from insurance when measured in wealth-equivalent units.

Part (b) Adverse selection:

(i) (2 points) The equation for the insurance company's zero expected profit line if the customer happens to be the high risk type is

$$0.5\ W_B + 0.5\ W_G = 0.5 * 1 + 0.5 * 4 = 2.5, \qquad \text{or} \qquad W_B + W_G = 5$$

(ii) (2 points) The equation for the insurance company's zero expected profit line when the customer happens to be the low risk type is

$$0.25\ W_B + 0.75\ W_G = 0.25 * 1 + 0.75 * 4 = 3.25, \qquad \text{or} \qquad W_B + 3\ W_G = 13$$

(iii) (4 points) In the separating equilibrium, the insurance contract available to be selected by the high risk type will offer them full insurance at the fair price corresponding to the probability of 0.5 of loss (again from class results). Therefore it will have $W_B = W_G$ along the zero expected profit line in (i), or $W_B = W_G = 2.5$. The resulting utility of the high risk type is $\ln(2.5) = 0.916$.

(iv) (12 points) In the separating equilibrium, the insurance contract available to be selected by the low risk type will be on the zero expected profit line in (ii), and such that the high risk type finds it exactly a matter of indifference to take this contract or the one in (iii). (The indifference can be broken to keep out the high risk type by a negligible change in the contract.) Again all this follows from the class results. The equation for this indifference is

$$0.5\ \ln(W_B) + 0.5\ \ln(W_G) = \ln(2.5)$$

or

$$W_B\ *\ W_G = (2.5)^2 = 6.25$$

Along the zero profit line, $W_B = 13 - 3\ W_G$. So we want

$$(13 - 3\ W_G)\ W_G = 6.25, \qquad \text{or} \qquad 3\ (W_G)^2 - 13\ W_G + 6.25 = 0$$

This yields two solutions, $W_G = 3.7826$ and then $W_B = 1.6526$, or $W_G = 0.5333$ and then $W_B = 11.4$. Of these, only the first makes economic sense – the second would take the low-risk types to an extreme outcome in the wrong direction.

The resulting expected utility of the low risk type is

$$0.25\ \ln(1.6526) + 0.75\ \ln(3.7826) = 1.1233$$

and the sure amount of wealth that would yield the same utility is $\exp(1.1233) = 3.075$.

(v) (9 points) The low risk type customer's expected utility where he gets no insurance at all is

$$0.25\ \ln(1) + 0.75\ \ln(4) = 1.0397$$

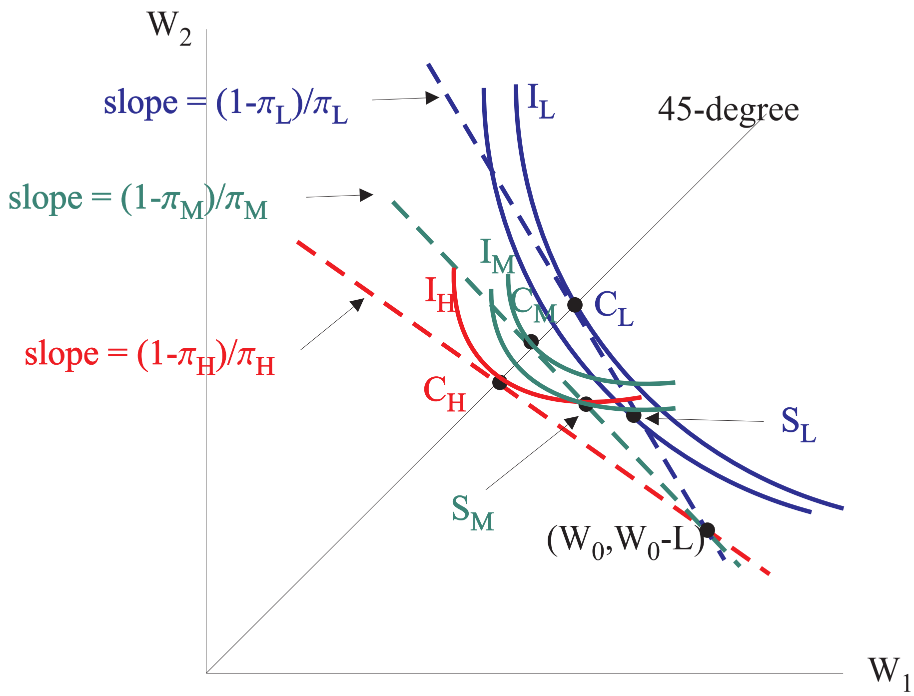
and the equivalent sure wealth is 2.828.

In the Rothschild-Stiglitz separating equilibrium, we found above that the expected utility 1.1233 and the equivalent sure wealth amount is 3.075.

If type were perfectly observable in the market and contracts could be conditioned on this, the low risk type would get full insurance at their fair price, so $W_B = W_G$ along the zero profit line in (ii), yielding $W_B = W_G = 3.25$ and the resulting utility would be $\ln(3.25) = 1.1786$.

Additional information: Thus the information asymmetry costs the low risk type 41.47% of the wealth-equivalent gain that is conceivable in the ideal hypothetical "first-best." (Calculation: $(3.25 - 3.075)/(3.25 - 2.828) = 0.4487$.) Note that the high-risk type continues to get full insurance at his fair price so does not suffer from the information asymmetry.

Question 2: (30 points)



If types were publicly observable and contracts could be conditioned on type, then each type would get full insurance on the actuarially fair insurance budget line, so contracts would be C_H, C_M and C_L. In the fully separating equilibrium illustrated, the high-risk types continue to get C_H. The middle-risk types get contract S_M that lies on their fair line but just prevents the H-types from taking it, and then the low-risk types get contract S_L that lies on their fair line and just prevents the M-types from taking it instead of S_M. Because the indifference curves of successively higher risk types are flatter (the Mirrlees-Spence property), this also ensures that the H-types prefer their own contract to the contract S_L.