

Question 1: (70 points)

In this question you are asked to solve special examples of insurance with moral hazard and adverse selection, following the methods developed in class quite closely. This is to reinforce your understanding of the somewhat intricate concepts and techniques. You will find the questions relatively straightforward if you follow the steps given; otherwise they can get quite difficult. Part (a) follows Slides 16 (moral hazard) and Part (b) follows Slides 15 (adverse selection). In Part (a) subparts (iv) and (v), and in Part (b) subparts (iii) and (iv), you do not have to prove your answers from scratch; you should use the results established in class, but should state what the relevant results are in each of these subparts.

Part (a) is about moral hazard and part (b) about adverse selection. The following information is common to the two parts. There are two states G and B, where G is the no-loss state and B the loss state. The customer has initial wealth $W_0 = 4$, and will suffer loss $L = 3$ if state B occurs. The customer is an expected utility maximizer with a logarithmic utility-of-consequences function over wealth, so if his final wealth levels (consumption) in the two states are W_G and W_B , and the probability of loss is π , his expected utility is

$$EU = (1 - \pi) \ln(W_G) + \pi \ln(W_B).$$

Note that the probability π will differ in different situations below, and the logarithms are natural (to base e , not 10). The insurance company is risk-neutral and wants to maximize expected profit, but perfect competition in the insurance market keeps its expected profit equal to zero.

Insurance contracts will be characterized by the amounts of wealth (W_G, W_B) that the customer will end up with in the two states. (Thus, if P is the premium and C the coverage, $W_G = W_0 - P$ and $W_B = W_0 - L - P + C$; but I urge you to forget about P and C and just work with W_B and W_G , that is, follow the Arrow-Debreu approach.)

Part (a)

The customer can affect the probability of loss by exerting effort. There are two effort levels, a bad one that has zero cost to the consumer in utility terms, and a good one that has cost k to the customer in utility terms. If the customer makes bad effort, the probability of loss is 0.5; if the customer makes good effort, the probability of loss is 0.25.

(i) (6 points) Find the inequality linking W_G , W_B and k such that the customer will make good effort if the inequality holds and bad effort if the opposite inequality holds. Assume that the customer will make good effort in the borderline case where the relation holds with exact equality.

(ii) (2 points) Find the equation for the insurance company's zero expected profit line when the customer makes bad effort.

(iii) (2 points) Find the equation for the insurance company's zero expected profit line when the customer makes good effort.

(iv) (5 points) Find the local optimum, namely the values of (W_G, W_B) that maximize the customer's expected utility subject to the insurance company's zero-profit constraint, when the customer is making bad effort, and find the customer's expected utility at this point.

(v) (8 points) Find the local optimum, namely the values of (W_G, W_B) that maximize the customer's expected utility subject to the insurance company's zero-profit constraint, when the customer is making good effort, and find the expression (as a function of k) for the customer's expected utility in this situation.

(vi) (8 points) Prove that the optimum in (v) is the global optimum if $k < 0.207$ approximately.

(vii) (9 points, 3 for each case of no insurance, the optimal incentive-compatible contract, and the hypothetical ideal) Now suppose $k = 0.1$. Find the customer's expected utility in three situations: first where he gets no insurance at all, second where he gets the globally optimal contract, and third, the hypothetical ideal optimum where effort is observable, and the customer is given full insurance at the statistically fair price conditional on his making good effort. In each of these three situations, find the magnitude of sure wealth that will yield the customer the same utility.

Part (b)

Here the customer may be innately more or less risky. The high risk types has probability of loss equal to 0.5; the low risk type has probability of loss equal to 0.25. The customer's type is private information. You are asked to consider a Rothschild-Stiglitz separating equilibrium.

(i) (2 points) Find the equation for the insurance company's zero expected profit line if the customer happens to be the high risk type.

(ii) (2 points) Find the equation for the insurance company's zero expected profit line when the customer happens to be the low risk type.

(iii) (4 points) What insurance contract intended for the high risk type will be offered in the equilibrium? What is the resulting expected utility?

(iv) (12 points) What insurance contract intended for the low risk type will be offered in the equilibrium? What is the resulting expected utility? What sure amount of wealth would yield the same utility to this customer?

(v) (9 points, 3 for calculation of each case) Compare the low risk type customer's expected utility and the equivalent sure wealth amounts in three situations: first where he gets no insurance at all, second where he gets the Rothschild-Stiglitz separating equilibrium contract, and third, the hypothetical ideal optimum where type is observable, and the customer is given full insurance at the statistically fair price for the low type.

Question 2: (30 points)

Consider the Rothschild-Stiglitz model of insurance with adverse selection and three unobservable types, high-risk, medium-risk, and low-risk, with loss probabilities $\pi_H > \pi_M > \pi_L$ respectively. Describe, and illustrate in a figure, the separating equilibrium that might exist. (IMPORTANT: The π_M here is the probability of loss for a third, middle-risk type in its own right, not the population-average over high-risk and low-risk types of Slides 16.)