

ECO 317 – Economics of Uncertainty – Fall Term 2009
Problem Set 7 – Due December 17

Question 1: (60 points)

This question closely follows the methods and steps of Lecture Note 19. Use the parallel to get ideas on how to proceed and to interpret what you are doing.

You are Oceania's Minister for Peace, and it is your job to purchase war materials for your ministry. Your experts have calculated that the country's gross benefit from quantity Q of these materials is $2Q^{1/2}$, measured in monetary units. So if you pay the total amount of money M for this purchase, the net benefit is $2Q^{1/2} - M$.

There is just one supplier, Baron Myerson's Armaments (BMA). You do not know BMA's cost of production. (The cost is interpreted as inclusive of the normal return to capital, as in the usual definition of economic cost.) You know that their cost per unit of output is constant (they have constant returns to scale), and the unit cost could take any of three values $c_1 < c_2 < c_3$, with respective probabilities π_1, π_2, π_3 . In what follows, we will say that BMA is of type i if its cost is c_i , for $i = 1, 2, 3$.

In the past, your ministry has used two kinds of purchase contracts: cost plus and fixed price. Cost plus contracts reimburse the supplier's cost. But this creates an incentive for BMA to inflate costs with items that are really perquisites and the like, and you are not able to tell the difference between true costs and this padding. A fixed price contract gives BMA the correct incentive to minimize cost, but they then keep all the profit, so you worry that your ministry may be giving away too much; it might have been possible to beat BMA down to accepting less. You have just read an article by Baron and Myerson (*Econometrica* 1982), and realize that it tells you how to design a new type of contract optimally.

You offer a menu of three possibilities: "Supply us quantity Q_i , and we will pay you money M_i ," for $i = 1, 2$, and 3. The idea is that if BMA is of type i , it should find contract i most profitable (the incentive compatibility or IC constraint); you can assume that if another contract is exactly as profitable, BMA will break its indifference by choosing i . And contract i should give BMA of type i non-negative profit (the participation or PC constraint).

(a) (5 points) Write down an expression for the profit of type- i BMA when it supplies quantity Q , and is paid M . Interpret $c_1 < c_2 < c_3$ as the single-crossing condition.

(b) (6 points) Write down the six incentive compatibility constraints and three participation constraints on your choice of (Q_i, M_i) . When writing the participation constraints, assume that the incentive constraints also hold, so that type i participates with the contract (Q_i, M_i) .

(c) (3 points) Write down the expression for your expected net benefit. This is what you want to maximize.

Now your problem is to choose the three Q_i and the three M_i to maximize expected net benefit subject to the incentive compatibility and participation constraints. Proceed as follows. In parts (d)-(g), assume that $Q_1 > Q_2 > Q_3$. You will be asked about this assumption in part (h).

(d) (12 points) Consider the “relaxed” maximization problem where only three constraints are imposed: the IC constraints for type 2 to prefer contract 2 over contract 3, and for type 1 to prefer contract 1 over contract 2, and the PC constraint for type 3; the other six constraints are not imposed. Use the three constraints that are imposed to derive lower bounds on your feasible choices of M_1 , M_2 , M_3 in terms of c_1 , c_2 , c_3 and Q_1 , Q_2 , Q_3 . (Note that two or more of the c ’s and Q ’s may appear in the expression for the lower bound for each of the M ’s.) Hence prove that all three of the constraints you have selected (two IC and one PC) will be binding at the optimum of the relaxed problem.

(e) (12 points) Then prove that the other six conditions (the four ICs and two PCs that were left out) are automatically satisfied. (So that, provided to the order assumption $Q_1 > Q_2 > Q_3$ holds, the solution to the relaxed problem will also be the solution to the full problem.)

(f) (2 points) Substitute out for the M_i to express your objective function of the relaxed problem in terms of the three Q_i only.

(g) (10 points) Write down the first-order conditions for the maximization, and solve them for the Q_i , to get the solution to the relaxed problem.

(h)(5 points) Show that the assumption made above, $Q_1 > Q_2 > Q_3$, will hold at the optimum of the relaxed problem if

$$\frac{c_3 - c_2}{c_2 - c_1} > \frac{\pi_1 \pi_3}{\pi_2}$$

(i) (5 points) What general principles of screening do your results in parts (g) and (h) illustrate? How does the optimal contract relate to the cost plus and fixed price contracts?

Question 2: (40 points)

The owner of a firm (the principal) hires a manager (the agent). If the agent chooses effort level x , the profit of the firm will be $y = x + \epsilon$, where ϵ is a random variable with mean $E[\epsilon] = 0$ and variance $V[\epsilon] = v$.

The owner offers the prospective manager a contract, which in this question will be assumed to be linear for simplicity of the analysis. The total compensation w of the manager will consist of a fixed wage h and a share s in the gross profit of the firm, so $w = h + s y$.

The utility functions of the owner (principal) and the manager (agent) are respectively

$$U_P = E[y - w] - \frac{1}{2} \alpha_P V[y - w],$$

and

$$U_A = E[w] - \frac{1}{2} \alpha_A V[w] - \frac{1}{2} k x^2.$$

The agent’s outside opportunity utility is U_A^0 .

Part (a):

- (i) (4 points) Using the substitutions

$$w = (h + s x) + s \epsilon, \quad y - w = -h + (1 - s) x + (1 - s) \epsilon,$$

express $E[w]$, $V[w]$, $E[y - w]$ and $V[y - w]$ as functions of x , h , and s .

- (ii) (2 points) Hence express U_P and U_A as functions of x , h and s .

Part (b):

(i) (6 points) First suppose the ideal case where the amount of effort x the manager exerts and the gross profit y are both publicly observable or verifiable. Then a contract can specify the x and the parameters h and s of the linear compensation function. The owner chooses x , h and s to maximize his utility U_P subject to the manager's participation constraint $U_A \geq U_A^0$. Show that the solution is

$$x = \frac{1}{k} \quad \text{and} \quad s = \frac{\alpha_P}{\alpha_A + \alpha_P}.$$

(ii) (4 points) Explain why these are independent of U_A^0 . What part of the contract does depend on U_A^0 ?

Part (c):

(i) (8 points) Now suppose x is not verifiable and therefore cannot be specified in the contract. The contract only specifies h and s . (The owner can calculate what x the manager will choose in response to the contract, but cannot prove this to a court.) Given h and s , show that the manager's effort choice will be

$$x = \frac{s}{k}.$$

(ii) (2 points) Express the resulting utilities U_P and U_A as functions of h and s .

(iii) (10 points) The owner chooses h and s to maximize U_P subject to $U_A \geq U_A^0$. Show that the solution is

$$s = \frac{1 + \alpha_P k v}{1 + (\alpha_P + \alpha_A) k v}.$$

(iv) (4 points) Is this value of s larger or smaller than that in Part (b)(i) above? Explain the intuition behind this comparison.