ECO 317 – Economics of Uncertainty – Fall Term 2009
Notes for lectures
2. The Expected Utility Theory of Choice Under Risk

1 Basic Concepts

The individual (usually consumer, investor, or firm) chooses action denoted by a, from a set of feasible actions A.

The set of states of the world is denoted by S, and probabilities are defined for events in S. Let $\pi(s)$ denote the probability of the elementary event s. Here we take these probabilities to be exogenous.

Actions can lead to different consequences in different states of the world. Let C denote the set of consequences, and $F : A \times S \mapsto C$ denote the consequence function, so $c = F(a, s)$ is the consequence of action a in state s.

For example, an investor's action might be a vector of proportions of the initial wealth that he puts into different kinds of assets. Then, depending upon the state of the world, e.g. interest rates rise and the dollar goes down, so bonds do badly but foreign stocks do well in dollar terms, there will be a consequence, namely his final wealth.

First suppose C is finite, say $C = \{c_1, c_2, \ldots c_n\}$, and write $p_i(a)$ for the probability that action a yields consequence c_i, that is,

$$p_i(a) = Pr\,\{\, s \,|\, F(a, s) = c_i \,\}\,.$$

Of course $p_i(a)$ may equal zero for some i.

In many economic applications, the set of consequences C is an infinite continuum of consumption quantities, or amounts of income or wealth. But the same idea has the obvious generalization. Each action induces a cumulative distribution function or a density function on C, e.g. we can define $p(w; a)$ such that $p(w; a)\,dw$ is the probability that the action a yields wealth between w and $w + dw$. We will usually develop the theory for consequence sets that are finite, but use it in the more general contexts without going into the rigorous (pedantic?) math of this.

Thus each action induces a probability distribution over C. Call such a probability distribution a *lottery.* The set of feasible actions will map into a set of feasible lotteries.

Different consequence functions may yield equivalent lotteries. Example: The state space is the set of possible outcomes of the roll of a die: $\{1, 2, 3, 4, 5, 6\}$. The consequence space is a set of prizes, {\$1,\$2,\$3}. The actions are bets. Bet 1, labeled a_1, yields \$1 if the die shows 1, 2, or 3, \$2 if it shows 4 or 5, and \$3 if it shows 6. Bet 2, labeled a_2, yields \$1 if the die shows an even number, \$2 if it shows 1 or 5, and \$3 if it shows 3. Then the two actions induce the same lottery, namely: the probability of getting \$1 is 1/2, that of getting \$2 is 1/3, and that of getting \$3 is 1/6.

In orthodox economic theory (we will consider some newer ideas such as framing and reference points later), a decision maker cares only about consequences and their probabilities. Therefore we can think of the choice problem as one of choosing among a set of feasible lotteries.

We will denote a lottery by a letter symbol, usually L with some label. More fully spelled out, it is a vector of probabilities of all the consequences $\{c_1,\, c_2,\, \ldots\, c_n\}$ in the correct order: $(p_1,\, p_2,\, \ldots\, p_n)$. Some of the p_i may be zero, and they sum to 1:

$$p_i \geq 0 \quad \text{for all } i; \qquad \sum_{i=1}^{n} p_i = 1\,.$$

Therefore we have a geometric representation: a lottery $L = (p_1,\, p_2,\, \ldots\, p_n)$ can be represented by a point on the unit simplex in n-dimensional space.

If many of the p_i in a lottery are zero, it may be simpler to write the lottery by listing the consequences that have non-zero probabilities, along with those probabilities, for example as $L = (c_1,\, c_3,\, c_7\,;\, p_1,\, p_3,\, p_7)$, where now p_1, p_3, and p_7 are positive and sum to 1.

Much of the theory can be illustrated using the case $n = 3$. Here an even simpler geometric representation, which we will call the *probability triangle diagram,* is available. Order the consequences so that C_1 is the worst and C_3 is the best for our decision-maker. In a two-dimensional diagram (Figure 1), show the probability p_1 of the worst consequence on the horizontal axis and the probability p_3 of the best consequence on the vertical axis. The point representing the lottery then lies in (or on the boundary of) the triangle given by $p_1 \geq 0$, $p_3 \geq 0$ and $p_1 + p_3 \leq 1$. The horizontal and vertical coordinates of the point show p_1 and p_3 as usual, whereas p_2 can be found implicitly as the horizontal (or vertical) distance of the point from the line $p_1 + p_3 = 1$. One of the probabilities is zero along each side of the triangle; for example along the long side (line $p_1 + p_3 = 1$), we have $p_2 = 0$. The extreme cases of no uncertainty correspond to the vertices of the triangle; for example, the origin is the "degenerate lottery" with $p_1 = p_3 = 0$ so $p_2 = 1$.

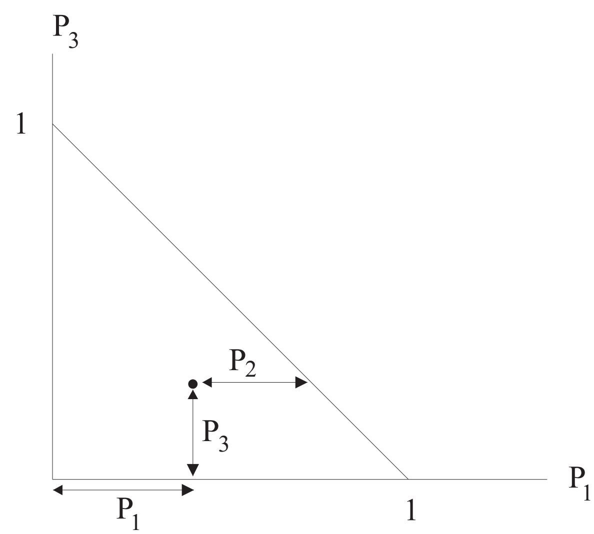

Figure 1: The probability triangle diagram

2 Composite or compound lotteries

These are lotteries whose prizes are themselves lotteries. Write a lottery that yields a prize consisting of lottery L^1 with probability p_1, a prize consisting of lottery L^2 with probability p_2, etc. using a self-evident notation:

$$L = \{L^1,\, L^2 \ldots L^k;\, p_1,\, p_2,\, \ldots p_k\}\,. \tag{1}$$

Suppose each of these lotteries is simple:

$$L^j = (q_1^j,\, q_2^j,\, \ldots\, q_n^j)\,. \tag{2}$$

The probability that you will get consequence c_i if you own the lottery L is then given by

$$Pr(c_i \,|\, L) = \sum_{j=1}^{k}\; Pr(c_i \,|\, L^j)\; Pr(L^j \,|\, L) = \sum_{j=1}^{k}\; q_i^j\; p_j\,. \tag{3}$$

Let us check that these probabilities sum to 1 when i ranges from 1 to n:

$$\begin{aligned} \sum_{i=1}^{n} \left\{ \sum_{j=1}^{k}\; q_i^j\; p_j \right\} \;&=\; \sum_{j=1}^{k} p_j \left\{ \sum_{i=1}^{n}\; q_i^j \right\} \\ &=\; \sum_{j=1}^{k} p_j * 1 = \sum_{j=1}^{k} p_j = 1\,. \end{aligned}$$

Making such checks is a good idea to improve one's mastery of the techniques and to avoid errors. In the future I will leave them as exercises.

A general presumption of standard microeconomic theory is that people care only about what they finally get to consume, and not by the path or process by which they arrived at this consumption vector. The natural analog in the case of uncertainty is that people should care only about the essential properties of any lottery or combination of lotteries they might hold, namely the probabilities of all the consequences. Specifically, they should be indifferent between a compound lottery (1) consisting of the constituent simple lotteries (2) on the one hand, and a single simple lottery whose probabilities are given by (3) on the other hand. This is sometimes called the *compound lottery axiom* or the *reduction of compound lotteries*.[1]

This axiom will be violated if the decision-maker likes or dislikes the process by which uncertainty is resolved; for example he may enjoy the suspense that builds up as a lottery yields a prize that is another lottery, or he may find it worrying. Some recent research considers departures of this kind from the standard theory, but we will not go into that.

Given the compound lottery axiom, we can show compound lotteries in the probability triangle diagram by showing their equivalent simple lotteries. Given any two lotteries L^1 and

[1]The textbook (p. 28) considers a different kind of compounding, where additional uncertainty is added, and a risk-averse consumer dislikes this. We are leaving all the probabilities of all the consequences the same in the simple lottery as they were in the original compound lottery.

L^2, and any probability p, consider the compound lottery $L^3 = \{L^1,\, L^2; p, (1-p)\}$. Write the probabilities of consequences 1 and 3 under L^j, using the above notation as $(q_1^j,\, q_3^j)$ for $j = 1, 2$. Then the probabilities of these consequences under L^3 are $(p\, q_1^1 + (1-p)\, q_1^2,\, p\, q_3^1 + (1-p)\, q_3^2)$. Therefore the point in the probability triangle diagram, the point that represents L^3 is on the straight line joining $(q_1^1,\, q_3^1)$ and $(q_1^2,\, q_3^2)$, dividing the distance in proportions $p : (1-p)$. If p is close to 1, then L^3 is close to L^1; if p is close to zero, L^3 is close to L^2.

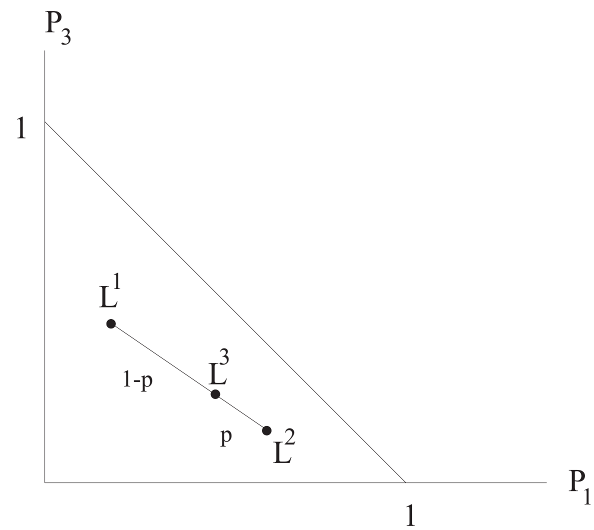

Figure 2: Compound lottery in probability triangle diagram

3 Indifference maps and expected utility

Again proceeding under the assumption of the compound lottery axiom, the consumer's preferences can only depend on the consequences $(c_1,\, c_2,\, \ldots\, c_n)$ and their respective probabilities $(p_1,\, p_2,\, \ldots\, p_n)$. In fact, when the list of consequences is comprehensive, and we are comparing different lotteries with different probabilities, we can just focus on these probabilities.

In general one might think that the preferences can be represented by a set of indifference curves, or an indifference map, in our probability triangle diagram of Figure 1. As usual, we can attach numbers to these indifference curves, where a curve corresponding to a higher level of preference gets a bigger number. This is a utility function, which can be written with the consequences and probabilities as its arguments:

$$F(c_1,\, c_2,\, \ldots\, c_n;\, p_1,\, p_2,\, \ldots\, p_n)\,.$$

The indifference map then consists of the contours $F =$ constant. However, such a general function is too unwieldy, and some special cases acquire special interest.

The case most commonly used is one where F has the form

$$p_1\; u(c_1) + p_2\; u(c_2) + \ldots + p_n\; u(c_n)\,. \tag{4}$$

In other words, there is a utility function u defined over consequences, and a lottery is evaluated by the mathematical expectation or expected value of this utility. The underlying u function is sometimes called a Bernoulli utility function or a von Neumann-Morgenstern utility function after the pioneers of this idea, and the overall expression above (4) is called *expected utility* of the lottery; write it as $EU(L)$.

Recall that a "degenerate" lottery yields only one consequence with probability 1; the probabilities of all other consequences are zero for this lottery. For a degenerate lottery $L^{(6)}$ yielding the consequence 6 with certainty, for example, expected utility is just $EU(L^{(6)}) = 1 * u(c_6) = u(c_6)$. So we can think of the Bernoulli utilities as the utilities of consequences, or as expected utilities of degenerate lotteries, whichever is better in any specific instance.

Because the functional form of $EU(L)$ in (4) is a very special case of the general function F, we cannot hope to represent any and all types of preferences over lotteries in this way. Just what restriction on preferences is implied by the expected utility form? We can see this in the probability triangle. Note that with three consequences, a contour of expected utility is given by

$$p_1\ u(c_1) + p_2\ u(c_2) + p_3\ u(c_3) = \text{constant} = k,\ \text{say},,$$

or

$$p_1\ u(c_1) + (1 - p_1 - p_3)\ u(c_2) + p_3\ u(c_3) = k\,,$$

or

$$p_3\ [u(c_3) - u(c_2)] - p_1\ [u(c_2) - u(c_1)] = k - u(c_2)\,. \tag{5}$$

Since in the probability triangle diagram the consequences are a fixed and exhaustive list while the probabilities vary, the numbers $u(c_i)$ are constants independent of the p_i. And since we are labeling the consequences so that c_1 is the worst and c_3 is the best, we must have $u(c_1) < u(c_2) < u(c_3)$. Therefore $[u(c_3) - u(c_2)]$ and $[u(c_2) - u(c_1)]$ are both positive constants. Therefore (5) is the equation of a straight line, with positive slope $[u(c_2) - u(c_1)]/[u(c_3) - u(c_2)]$. This slope is the same for all indifference curves (independent of the constant k); therefore the indifference map must be a family of parallel straight lines. And since expected utility must increase when the probability of the best consequence increases and/or the probability of the worst consequence falls, expected utility must increase as we move to the North-West from one indifference curve to another. All this is shown in Figure 3.)

4 Foundations of expected utility

We can also interpret the parallel indifference curve property in terms of the basic preference relationship among lotteries. This is a binary relation $\succeq$. For two lotteries L^1 and L^2, the statement $L^1 \succeq L^2$ means that the decision-maker regards L^1 at least as good as L^2. The opposite relation is $\preceq$.

Recall the "Axioms of Rational Choice" of consumer theory in the absence of uncertainty that enabled you to use a utility function in ECO 310, e.g. Nicholson's Microeconomic Theory ninth edition Chapter 3, pp.69–70. The first is *completeness,* which says that the consumer can always compare any two situations. Here it means that for any two lotteries L^1 and L^2,

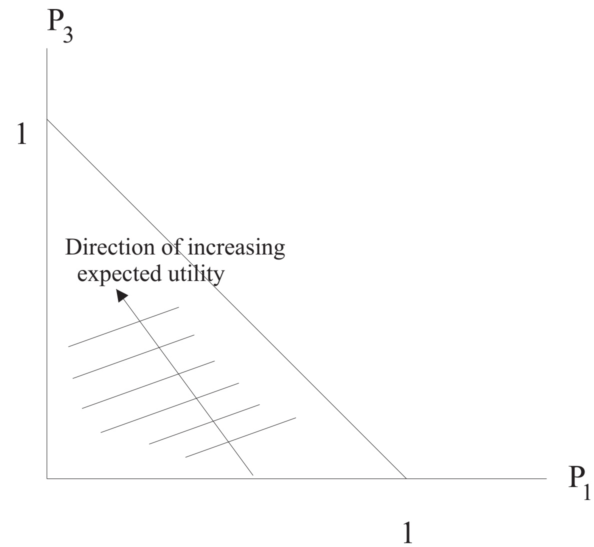

Figure 3: Indifference curves of expected utility

at least one of $L^1 \succeq L^2$ and $L^1 \preceq L^2$ must be true. If both are true, then the decision-maker regards L^1 as indifferent to L^2, written $L^1 \sim L^2$. If $L^1 \succeq L^2$ is true but $L^1 \preceq L^2$ is false, then L^1 is strictly preferred to L^2, written $L^1 \succ L^2$. And the other way round.

Then we have *transitivity;* if $L^1 \succeq L^2$ and $L^2 \succeq L^3$ are both true, then $L^1 \succeq L^3$ must be true. Finally we have *continuity;* if $L^1 \succ L^2$ and L^3 is sufficiently close to L^2 (here it means the probability vectors of L^2 and L^3 are sufficiently close), then it will be true that $L^1 \succ L^3$.

The same three axioms will enable us to establish the existence of a general utility function like the F above. But we need another axiom to have the expected utility form (4).

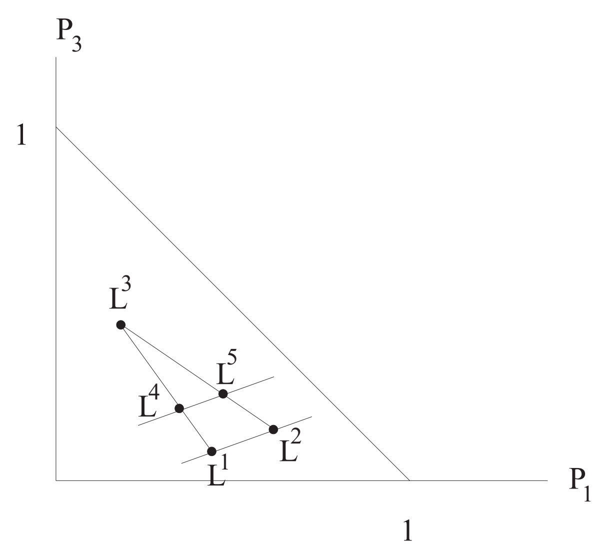

Figure 4: Independence axiom

To find and interpret this extra condition, consider Figure 4. I have shown two lotteries

L^1 and L^2 on one indifference curve and therefore indifferent to each other, and any third lottery L^3. (In this figure L^3 is preferred to either of L^1 and L^2, but that plays no role in the argument; as a useful exercise to reinforce your understanding, you should construct the figure and the argument when $L^3 \prec L^1 \sim L^2$.) Now take any probability p, and consider the compound lotteries

$$L^4 = (L^1, L^3; p, 1-p), \qquad L^5 = (L^2, L^3; p, 1-p)\,. \tag{6}$$

Recalling the procedure explained in Figure 2, L^4 is represented by its equivalent simple lottery, namely the point on the straight line joining L^1 to L^3 and dividing it in proportions $p : (1-p)$, and similarly for L^5 in relation to L^2 and L^3.

The proportions $p : (1-p)$ being the same in the two cases, the line joining L^4 to L^5 must be parallel to the line joining L^1 to L^2. But we took L^1 and L^2 to be indifferent to each other, so the line joining them is simply an indifference curve of expected utility. But if the indifference map represents expected utility contours, then indifference curves are a family of parallel straight lines. Therefore the line joining L^4 to L^5 must constitute an indifference curve as well. That is, L^4 and L^5 must be indifferent to each other.

So we have the result: If preferences can be represented by expected utility, then for any two mutually indifferent lotteries L^1 and L^2, any third lottery L^3, and any probability p, the lotteries L^4 and L^5 defined by (6) must be indifferent as well. A slightly different but equivalent way to put it is: If, in a compound lottery $(L^1, L^3; p, 1-p)$ we replace L^1 by another lottery L^2 that is indifferent to L^1, the resulting compound lottery $(L^2, L^3; p, 1-p)$ is indifferent to the original compound lottery.

If we move L^1 to a slightly higher indifference curve than L^2, then L^4 will move to a slightly higher indifference curve than L^5. This gives us the following property: If preferences can be represented by expected utility, then for any two lotteries L^1 and L^2 such that $L^1 \succeq L^2$, any third lottery L^3, and any probability p, the lotteries L^4 and L^5 defined by (6) must satisfy $L^4 \succeq L^5$.

The converse is also true. Give the name the *independence axiom* to the property that for any two lotteries L^1 and L^2 satisfying $L^1 \succeq L^2$, any third lottery L^3, and any probability p, the lotteries L^4 and L^5 defined by (6) must satisfy $L^4 \succeq L^5$. Then we have the

Expected Utility Theorem: If a decision-maker's preferences satisfy the independence axiom, together with the usual properties of completeness, transitivity, and continuity, then the preferences can be represented by expected utility.

The proof is complicated and I omit most of it, merely giving the construction of such a utility function in a slightly special context. This supposes that among all feasible lotteries, there is a best, say $\overline{L}$, and a worst, $\underline{L}$. If the set of all possible consequences is finite, then one need merely pick the best and the worst consequences. If the set is infinite, and open or unbounded at the extremes, then the procedure below needs to be modified. For those interested, the Herstein-Milnor article in the optional reading, or a fuller reading of those pages from Arrow's chapter which are to be skimmed in the required reading, give the complete proof.

Consider any lottery L, and compare it to the compound lottery $L^p = (\overline{L}, \underline{L}; p, 1-p)$ as p ranges from 0 to 1. This L^p must surely rank higher and higher in preferences as p increases.

For p close to zero, $L^p \approx \underline{L}$ so our original lottery L must be better than L^p. For p close to 1, $L^p \approx \overline{L}$ so our original lottery L must be worse than L^p. Therefore there must be one and only one p such that $L \sim L^p$. (The difficulties of the formal proof consist entirely of making all these statements precise and rigorous as is done in "real analysis" or "point set topology".) Define the utility of L to be this number p itself. So the utility function u we are trying to construct has $u(L) = p$. We check that it has the expected utility property.

First, consider all the pure consequences c_i. Think of each as a degenerate lottery, and let q_i be its utility according to this definition, that is q_i is the number such that c_i is indifferent to $(\overline{L}, \underline{L}; q_i, 1 - q_i)$. So $u(c_i) = q_i$.

Now suppose our lottery L, or its equivalent simple lottery, is expressed in terms of the consequences as $(c_1, c_2, \ldots c_n; p_1, p_2, \ldots p_n)$. The degenerate lottery c_1 is indifferent to $(\overline{L}, \underline{L}; q_1, 1 - q_1)$. Using the independence axiom in its interpretation of replacing one part of a compound lottery by something indifferent to it, we see that L is indifferent to the compound lottery

$$L' = (\overline{L}, \underline{L}, c_2, \ldots c_n; p_1\, q_1, p_1(1 - q_1), p_2, \ldots p_n)\,.$$

Next, c_2 is indifferent to $(\overline{L}, \underline{L}; q_2, 1 - q_2)$. By the independence axiom again, the lottery L' is indifferent to

$$(\overline{L}, \underline{L}, \overline{L}, \underline{L}, c_3, \ldots c_n; p_1 q_1, p_1(1 - q_1), p_2 q_2, p_2(1 - q_2), p_3 \ldots p_n)\,.$$

Proceeding in this way, our initial lottery L is finally indifferent to

$$(\overline{L}, \underline{L}, \overline{L}, \underline{L}, \ldots \overline{L}, \underline{L}; p_1 q_1, p_1(1 - q_1), p_2 q_2, p_2(1 - q_2), \ldots p_n q_n, p_n(1 - q_n))\,.$$

By the compound lottery axiom only the eventual consequences and their probabilities matter, so we can collect this into

$$(\overline{L}, \underline{L}; p_1 q_1 + p_2 q_2 + \ldots p_n q_n,\ p_1(1 - q_1) + p_2(1 - q_2) + \ldots + p_n(1 - q_n))\,,$$

or

$$(\overline{L}, \underline{L}; p_1 q_1 + p_2 q_2 + \ldots p_n q_n,\ 1 - (p_1 q_1 + p_2 q_2 + \ldots + p_n q_n))\,.$$

Therefore, by our definition of u,

$$\begin{aligned} u(L) &= p_1\, q_1 + p_2\, q_2 + \ldots + p_n\, q_n \\ &= p_1\, u(c_1) + p_2\, u(c_2) + \ldots + p_n\, u(c_n)\,, \end{aligned} \tag{7}$$

which is exactly the expected utility form. So henceforth instead of writing it as $u(L)$ we will write it as $EU(L)$.

5 Cardinal utility

If in (7) we replace the Bernoulli (or von Neumann-Morgenstern) utility function u of consequences by another function $\tilde{u}$ defined by

$$\tilde{u}(c) = a + b\ u(c)$$

for all consequences c, where a and b are constants, with $b > 0$, the expected utility of any lottery L becomes

$$\begin{aligned} E\tilde{U}(L) &= p_1\, \tilde{u}(c_1) + p_2\, \tilde{u}(c_2) + \ldots + p_n\, \tilde{u}(c_n) \\ &= p_1\, [a + b\, u(c_1)] + p_2\, [a + b\, u(c_2)] + \ldots + [a + b\, p_n\, u(c_n)] \\ &= a\, (p_1 + p_2 + \ldots + p_n) + b\, [\, p_1\, u(c_1) + p_2\, u(c_2) + \ldots + p_n\, u(c_n)\,] \\ &= a\, (p_1 + p_2 + \ldots + p_n) + b\, [\, p_1\, u(c_1) + p_2\, u(c_2) + \ldots + p_n\, u(c_n)\,] \\ &= a + b\; EU(L) \end{aligned}$$

Therefore for any two lotteries L^1 and L^2,

$$E\tilde{U}(L^1) > E\tilde{U}(L^2) \quad \text{if and only if} \quad EU(L^1) > EU(L^2)\,.$$

Therefore EU and $E\tilde{U}$ represent the same preferences. Thus the expected utility representation is not unique; the choice of the underlying Bernoulli utility function of consequences is arbitrary up to an increasing linear transformation. Roughly speaking, the "origin" and the "scale" of Bernoulli utility can be chosen arbitrarily.

In consumer choice theory under certainty, the choice of a utility function to represent a given set of preferences was even more arbitrary, namely up to an increasing transformation, linear or non-linear. For example, Cobb-Douglas preferences could be represented equally well by

$$\text{either} \quad X^{\alpha}\, Y^{\beta}, \qquad \text{or} \quad \alpha\; \ln(X) + \beta\; \ln(Y)\,.$$

A similar freedom to use nonlinear transformation of the Bernoulli utility function is not available under uncertainty, because the expected values of a non-linear transform is not any non-linear transform of the expected value. For example

$$p_1\; \ln(c_1) + p_2\; \ln(c_2) \neq \ln(\, p_1\, c_1 + p_2\, c_2\,)\,.$$

The two correspond to different degrees of risk aversion. An intuitive explanation follows; we will develop the concept of risk aversion in more detail later.

In consumer theory under certainty, only the ordering of utility numbers matters: an indifference curve higher up in the preferences should be assigned a larger number, but how much larger is immaterial and has no significance. Therefore that utility is said to be *ordinal*. Under uncertainty, and specifically under the expected utility theory, the size of differences in utility numbers matters, at least up to a choice of scale. For example, if the consequences are magnitudes of wealth with $c_1 < c_2 < c_3$, then whether $[u(c_2) - u(c_1)]$ is bigger or less than $[u(c_3) - u(c_2)]$ matters for whether

$$u(c_2) > \text{or} < \tfrac{1}{2}\, u(c_1) + \tfrac{1}{2}\, u(c_3)\,,$$

and therefore for whether the decision-maker would prefer to have c_2 for sure, or a 50:50 gamble between c_1 and c_3. Therefore a non-linear transformation of the $u(c)$ numbers, which could change $[u(c_2) - u(c_1)]$ and $[u(c_3) - u(c_2)]$ in quite different ways, can affect this comparison and therefore would not represent the same underlying attitude toward risk.

Since the magnitudes of utility numbers matter (up to the choice of origin and scale), the utility function that goes into expected utility calculations is said to be *cardinal*. Some people think that this gives utility a tangible quality, and speak of "measuring" utility much as one might measure the quantity of corn or steel. I think this is a mistake. Expected utility theory is an economists' construct, to enable us to represent preferences numerically and therefore to facilitate calculations about choice and later about equilibrium. The utility numbers constructed in this theory don't exist "out there"; there is no point in trying to attach them any physical or tangible significance.

6 Limits to expected utility theory

Indifference curves being parallel straight lines in the probability triangle diagram (or a higher dimensional simplex), and the corresponding property of preferences, namely independence axiom, is a restriction. Not all preferences, even fully rational ones (satisfying completeness and transitivity), need satisfy this restriction. But is it a reasonable restriction in practice?

It may seem innocuous to replace one lottery by another which is indifferent to it, but in many experiments and observations actual behavior does run counter to the independence axiom. In the questionnaire posted in advance of this topic, Question 2 asked how you would choose between the lotteries A and B, where

A: \$2,500 with probability 0.33, \$2,400 with probability 0.66, \$0 with probability 0.01
B: \$2,400 for sure

Question 3 posed the hypothetical choice between C and D, where

\$2,500 with probability 0.33, \$0 with probability 0.67
\$2,400 with probability 0.34, \$0 with probability 0.66

If your behavior conformed to expected utility theory, you would choose A or B according as

$$0.33\ u(2500) + 0.66\ u(2400) + 0.01 u(0) > \text{ or } < u(2400)\,,$$

that is,

$$0.33\ u(2500) + 0.01 u(0) > \text{ or } < 0.34\ u(2400)\,.$$

In Question 6 you were asked to choose between two different lotteries: C yielding \$2,500 with probability 0.33 and \$0 with probability 0.67, and D yielding \$2,400 with probability 0.34 and \$0 with probability 0.66. If your behavior conformed to expected utility theory, you would choose C or D according as

$$0.33\ u(2500) + 0.67\ u(0) > \text{ or } < 0.34\ u(2400) + 0.66\ u(0)\,,$$

that is,

$$0.33\ u(2500) + 0.01 u(0) > \text{ or } < 0.34\ u(2400)\,.$$

The two criteria are the same! Therefore you would be choosing A in Question 5 if and only if you choose C in Question 6. Which of the choice combinations A, C or B, D you make would depend on your risk aversion, which is reflected in the function u. But regardless of that, you would choose one of these two combinations. If you choose either A, D or B, C, then your behavior is inconsistent with expected utility theory.

When I ran this experiment in the Fall 2007 class, the actual returns were: AC = 10, BC = 8, BD = 5, AD = 1. This is slightly better for expected utility theory than is usual in such experiments, but the sample size is too small to draw any strong conclusions.

The argument that many rational decision-makers would choose BC in such situations was made by Maurice Allais, and this became known as the Allais Paradox. Actually there is nothing paradoxical in the sense of irrationality. However, there are some other associated aspects that constitute stronger violations of the standard economic theory of rational choice.

In a couple of weeks we will discuss the Allais Paradox and some related departures from expected utility theory more fully. We will relate such behavior to the shape of the indifference map in the probability triangle diagram, and discuss somewhat more general classes of preferences and some of their consequences. That said, expected utility theory remains the basic approach to the analysis of economic and financial choices and equilibria, and we will follow it also.