ECO 317 – Economics of Uncertainty – Fall Term 2009
Notes for lectures
8. State Space Representation of Preferences

Thus far we have developed the theory of choice under uncertainty by defining preferences over lotteries. This kept the underlying states of the world hidden in the background. For developing and illustrating the theory of insurance, and for later work involving general equilibrium of trading risks, it helps to bring the states more explicitly into the picture.

This approach is a natural adaptation of the standard indifference curves and budget lines of consumer theory to the case of uncertainty. For geometric treatment, we have to consider only two states, but the theory can be done perfectly well with any number of states just as standard consumer theory illustrated using two-good figures was valid for any number of goods.

We show on the two axes the amounts of final wealth of the consumer in the two states. The indifference map consists of contours of equal expected utility,

$$EU \equiv (1-\pi)\ u(W_1) + \pi\ u(W_2) = \text{constant}.$$

Figure 1 shows this. The contours have some properties specific to the present context.

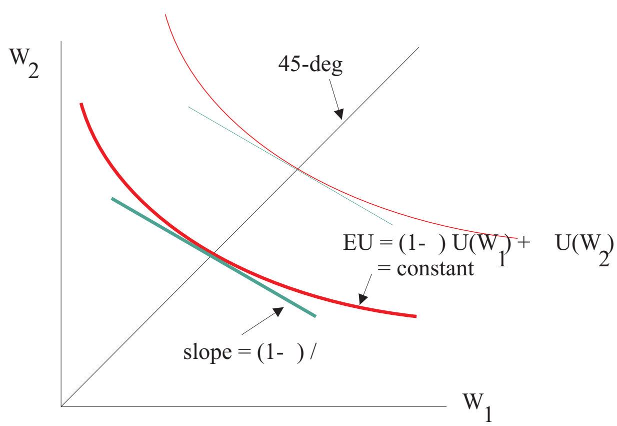

Figure 1: Contours of expected utility

First, the marginal rate of substitution is

$$- \left. \frac{dW_2}{dW_1} \right|_{EU\ =\ \text{constant}} = \frac{\partial EU / \partial W_1}{\partial EU / \partial W_2} = \frac{1-\pi}{\pi}\ \frac{u'(W_1)}{u'(W_2)}\,. \tag{1}$$

As we proceed down and to the right along any one indifference curve, W_1 increases and W_2 decreases. With a risk-averse individual, this lowers $u'(W_1)$ and raises $u'(W_2)$. Both contribute to a lowering of the right hand side in (1). Thus the indifference curve gets flatter; we have a convex indifference curve, showing dimishing marginal rate of substitution. In other words, the usual property of indifference curves in consumer theory corresponds to risk aversion when the commodities are wealths (or income or consumption or whatever is being considered in each context) in different states of the world.

Next, the 45-degree line gets particular significance. On all points of it, $W_1 = W_2$; the individual has equal wealth in the two states, and therefore faces no risk. (Of course he is not indifferent among all points on the 45-degree line; he clearly prefers points farther to the northeast.) At any point of the 45-degree line, we also have $u'(W_1) = u'(W_2)$, and the expression (1) for the marginal rate of substitution reduces to $(1-\pi)/\pi$, the probability ratio or the odds against the occurance of the loss. This will be useful for a lot of subsequent analysis and interpretation.

Finally, if the individual is very risk-averse, then u'' is large in magnitude. Therefore $u'(W_1)$ decreases rapidly as W_1 increases and $u'(W_2)$ increases rapidly as W_2 decreases. That is, each indifference curve changes slope rapidly as we proceed down and to the right along it; its convexity or curvature is high. Conversely, if the individual has a low aversion to risk, the indifference curve is much less curved, that is, it is closer to being a straight line.

Figure 2 illustrates this by showing indifference maps for two individuals with different degrees of risk aversion. The thick and relatively flat set is for a person whose coefficient of relative risk aversion ρ equals 0.2, and the thin and more curved set is for a person with $\rho = 2$. A fully risk-neutral individual would have straight line indifference curves with slope equal to $(1-\pi)/\pi$.

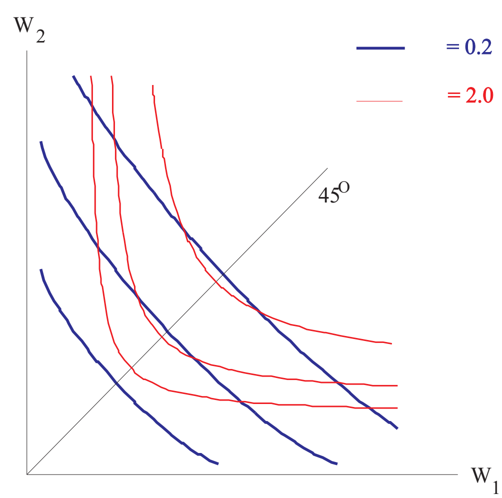

Figure 2: Different degrees of risk aversion

In the limit as $\rho \to \infty$, indifference curves become L-shaped with the corner of the L on the 45-degree line; thus an infinitely risk-averse person chooses to be on the 45-degree line, eliminating all risk. This is also very intuitive.