ECO 317 – Economics of Uncertainty – Fall Term 2009
Slides to accompany
20. Incentives for Effort - One-Dimensional Cases

1. Linear Incentive Schemes

Agent's effort x, principal's outcome y. Agent paid w.
$y = x + \epsilon$ where $\mathsf{E}[\epsilon] = 0$), $\mathsf{V}[\epsilon] = v$.
(Note: x is not random; it is chosen by the agent.)
Agent's outside opportunity utility U_A^0. In this job,

$$U_A = \mathsf{E}[w] - \tfrac{1}{2}\,\alpha\,\mathsf{V}[w] - \tfrac{1}{2}\,k\,x^2$$

Principal's utility $U_P = \mathsf{E}[y - w]$.

Hypothetical Ideal or First-Best

x verifiable. Principal chooses contract (x, w) to max

$$U_P = \mathsf{E}[y - w] = \mathsf{E}[x + \epsilon - w] = x - \mathsf{E}[w]\,,$$

subject only to the agent's participation constraint (PC)

$$U_A = \mathsf{E}[w] - \tfrac{1}{2}\;\alpha\;\mathsf{V}[w] - \tfrac{1}{2}\,k\;x^2 \geq U_A^0\,.$$

Obviously $\mathsf{V}[w] = 0$ and $\mathsf{E}[w]$ lowest to meet PC. Then

$$U_P = x - \tfrac{1}{2}\,k\;x^2 - U_A^0\,.$$

Optimal x from FOC $1 - \frac{1}{2}\,2\,k\,x = 0$, so $x = 1/k$.

Result

$$w = U_A^0 + \tfrac{1}{2}\,k\,x^2 = U_A^0 + \frac{1}{2\,k}\,,$$

$$U_A = U_A^0\,, \qquad U_P = \frac{1}{2\,k} - U_A^0\,.$$

Second-Best Linear Incentive Schedules

x unverifiable (y verifiable, but can't infer x precisely from y).

Consider linear (really, affine) contract with payment

$$w = h + s\,y = h + s\,(x + \epsilon) = (h + s\,x) + s\,\epsilon\,,$$

Then

$$\mathsf{E}[w] = h + s\,x\,, \qquad \mathsf{V}[w] = s^2\ \mathsf{V}[\epsilon] = s^2\ v\,,$$

and

$$U_A = h + s\,x - \tfrac{1}{2}\,\alpha\,v\,s^2 - \tfrac{1}{2}\,k\,x^2\,.$$

Agent chooses x to max this. FOC $s - k\,x = 0$, so $x = s/k$.

s is a measure of the implied "power of incentive".

Substituting for x, agent's maximized or "indirect utility" function:

$$U_A^* = h + s\,\frac{s}{k} - \tfrac{1}{2}\,\alpha\,v\,s^2 - \tfrac{1}{2}\,k\,\left(\frac{s}{k}\right)^2 = h + \frac{s^2}{2\,k} - \tfrac{1}{2}\,\alpha\,v\,s^2\,.$$

Principal's utility

$$U_P = \mathsf{E}[y - h - s\,y] = (1-s)\,x - h = (1-s)\,\frac{s}{k} - h\,.$$

The principal chooses contract (h, s) to max this,
subject to the agent's PC $U_A^* \geq U_A^0$. (IC used in choice of x).

$$h + \frac{s^2}{2\,k} - \tfrac{1}{2}\,\alpha\,v\,s^2 \geq U_A^0\,.$$

Obviously optimal to keep h as low as feasible:

$$h = U_A^0 - \frac{s^2}{2\,k} + \tfrac{1}{2}\,\alpha\,v\,s^2\,.$$

and

$$\begin{aligned} U_P &= \frac{s\,(1-s)}{k} + \frac{s^2}{2\,k} - \tfrac{1}{2}\,\alpha\,v\,s^2 - U_A^0 \\ &= \frac{s}{k} - \frac{s^2}{2\,k} - \tfrac{1}{2}\,\alpha\,v\,s^2 - U_A^0\,. \end{aligned}$$

Choosing s to maximize this, FOC

$$\frac{1}{k} - \frac{s}{k} - \tfrac{1}{2}\,\alpha\,v\,(2\,s) = 0\,;$$

$$s = \frac{1}{1 + \alpha\,v\,k}\,.$$

Intuition and interpretation:

[1] $0 < s < 1$. First-best risk-sharing would make $s = 0$,
but when x is unverifiable, moral hazard requires $s > 0$.
First-best effort incentive would be $s = 1$, but that puts
too much risk on agent. Second best balances these two.
The choice of h arranges split of surplus between parties.

[2] The higher is α, the lower is s.
When agent more risk-averse, giving more powerful incentive
makes his income too risky; must increase h to maintain PC.

[3] The higher is v, the lower is s. A high v means
less accurate inference of x from y. Powerful incentive wasted.

[4] Utilities in second-best optimum:

$$U_P = \frac{1}{2\,k\,(1+\alpha\,v\,k)} - U_A^0\,, \qquad U_A = U_A^0\,.$$

If

$$\frac{1}{2\,k\,(1+\alpha\,v\,k)} < U_A^0 < \frac{1}{2\,k}\,,$$

contract should be made under first best but not second-best.

[5] Order of magnitude from John Garen (JPE December 1994):

Data on large U.S. corporations during 1970-1988.

Median market value $2 billion 2×10^9, median variance $v \approx 2 \times 10^{17}$.

CEOs median income $1 million (1×10^6).

Coefficient of relative risk aversion 2, absolute $\alpha = 2 \times 10^{-6}$.

Then

$$\mathsf{E}[y] = x = s/k = 2 \times 10^9, \quad s = 1/(1 + \alpha\, v\, k) = 1/(1 + 4 \times 10^{11}\, k)\,.$$

Eliminating k between the two equations,

$$s = 1/(1 + 200\, s), \quad \text{or} \quad 200\ s^2 + s - 1 = 0\,, \quad s \approx 0.0683.$$

Actual values are much smaller, averaging 0.0142.

Other considerations can explain lower power.

2. Nonlinear Incentive Schedules

Agent chooses effort x, cost $K(x)$.
  Principal's outcome: Values y_i increasing, probabilities $\pi_i(x)$.
  Higher x shifts distribution FOSD to the right.
Utilities

$$EU_A = \sum_{i=1}^{n} \pi_i(x)\ u_a(w_i) - K(x)\,,$$

$$EU_P = \sum_{i=1}^{n} \pi_i(x)\ u_p(y_i - w_i)\,.$$

Ideal first-best

Contract (x, w_i) to max EU_P subject to $EU_A \geq U_A^0$.

$$\mathcal{L} = \sum_{i=1}^{n} \pi_i(x)\ u_p(y_i - w_i) + \lambda\ \left\{ \sum_{i=1}^{n} \pi_i(x)\ u_a(w_i) - K(x) - U_A^0 \right\}\,.$$

FOCs for payments w_j

$$\frac{\partial \mathcal{L}}{\partial w_j} = \pi_j(x) \ \Big[- u_p'(y_j - w_j) + \lambda \ u_a'(w_j) \ \Big] = 0 \,,$$

or, as in Arrow-Debreu theory (Handout 13 pp. 6-7):

$$\frac{u_p'(y_j - w_j)}{u_a'(w_j)} = \lambda \qquad \text{for all } j \,.$$

Moral hazard

Agent chooses unverifiable x to maximize EU_A. FOC

$$\frac{\partial EU_A}{\partial x} = \sum_{i=1}^{n} \ \pi_i'(x) \ u_a(w_i) - K'(x) = 0 \,.$$

Here assume that solution to FOC yields true optimum

That is actually problematic; more advanced treatments discuss this.

That FOC becomes the IC in principal's choice.

$$\mathcal{L} \;=\; \sum_{i=1}^{n} \pi_i(x)\; u_p(y_i - w_i) + \lambda \left\{ \sum_{i=1}^{n} \pi_i(x)\; u_a(w_i) - K(x) - U_A^0 \right\}$$
$$+\mu \left\{ \sum_{i=1}^{n} \pi_i'(x)\; u_a(w_i) - K'(x) \right\} .$$

FOCs for the w_j

$$\frac{\partial \mathcal{L}}{\partial w_j} = \pi_j(x)\; \Big[-\, u_p'(y_j - w_j) + \lambda\; u_a'(w_j) \Big] + \mu\; \pi_j'(x)\; u_a'(w_j) = 0\,,$$

or

$$\frac{u_p'(y_j - w_j)}{u_a'(w_j)} = \lambda + \mu\; \frac{\pi_j'(x)}{\pi_j(x)} \qquad \text{for all } j\,.$$

Then w_j high if $\pi_j'(x) \,/\, \pi_j(x) = d\, \ln\left[\pi_j(x)\right] \,/\, dx$ high.
Such states are most informative about slackening of effort.
So high payments in them give best incentives.

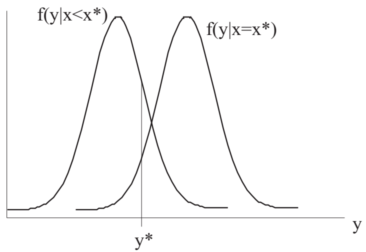
Special Case – Quotas

Choose threshold y^* and w_L, w_H in contract:

$$w(y) = \begin{cases} w_L & \text{if } y < y^*, \\ w_H & \text{if } y \geq y^*, \end{cases}$$

This works well if, for x slightly smaller than principal's optimal x^*,

$$\text{Prob}\{\, y \geq y^* \mid x \,\} << \text{Prob}\{\, y \geq y^* \mid x^* \,\}$$



Special case – Efficiency Wage

Repeated interaction. Agent's action observable with delay.

Contract: agent paid each period more than outside opportunity, but fired if he is ever caught shirking.

Example: Effort binary (good or bad). Cost of good C.

Outside wage in non-moral-hazard jobs W_0.

Contract: Suppose the agent is paid W when not detected shirking.

Probability of detection P. Discount factor δ.

Expected present value of cost of shirking

$$P\ (W - W_0)\ (\delta + \delta^2 + \delta^3 + \dots\ ) = P\ (W - W_0)\ \delta/(1-\delta)\,.$$

Keep this $\geq C$ to deter shirking. So "efficiency wage"

$$W \geq W_0 + \frac{1-\delta}{\delta}\ \frac{C}{P}\,.$$