ECO 352 – International Trade – Spring Term 2010
Week 4 Precepts – February 22
EFFECTS OF GROWTH ON A TRADING ECONOMY – QUESTIONS

Question 1:

Here we ask whether growth can harm a trading economy. To keep the math simple, we consider growth as an exogenous increase of endowment in a pure exchange economy.

The world of this question has two countries, France and Holland, and two goods, wine and cheese. France has an endowment only of Wine, and Holland has an endowment only of cheese. Preferences are identical and homothetic in this world, and the relative demands are given by

$$\frac{C}{W} = \left(\frac{P_C}{P_W} \right)^{-1/2}$$

in the obvious notation. You are told that these come from a utility function

$$U(C, W) = \frac{C\,W}{C + W}\,.$$

You are invited, in your advance preparation, to derive the relative demand function by maximizing utility subject to the budget constraint, but need not do so. We will go over it in the precept.

The question has two parts. In both parts, Holland's cheese endowment is 1. For brevity, write P for the relative price of cheese in terms of wine, P_C/P_W.

First suppose France's wine endowment is also 1. What is the equilibrium value of P? Use France's budget constraint to find its equilibrium consumption quantities of cheese and wine. Find France's resulting utility.

Now suppose France's wine endowment is 2. Repeat these calculations.

You should find that the growth in its endowment of wine has *lowered* France's utility. What is the economic intuition for this?

Question 2:

Here we consider the effects of growth on the supply response of an economy whose production is of the sector-specific capital kind.

In class on Thursday 2/18, we derived an elliptical PPF for a special example of such an economy. Let us generalize it slightly. Suppose production functions in the two sectors are

$$X = A_X\ \left(K_X\, L_X \right)^{1/2}, \qquad Y = A_Y\ \left(K_Y\, L_Y \right)^{1/2},$$

where X and Y are the output quantities, K_X and K_Y the capital endowment specific to the two sectors, L_X and L_Y are the labor allocations to the two sectors subject to the constraint imposed by the total endowment, namely $L_X + L_Y = L$, and A_X, A_Y are productivity parameters. Thus increases in K_X, K_Y and L represent growth through an increase in factor endowments, and increases in A_X, A_Y represent growth through technical progress.

Solving the production functions for L_X and L_Y, and substituting into the labor allocation equation, the equation of the PPF is

$$\frac{X^2}{(A_X)^2\ K_X} + \frac{Y^2}{(A_Y)^2\ K_Y} = L\,.$$

Given the relative price P_X/P_Y, the country's supply response is found at the point of tangency of the PPF with a line whose slope is the relative price.

(a) If A_X increases, how does the PPF shift? Compare the MRT's of the old and the new PPF's at points of equal Y. What do you conclude about the effect of the increase in A_X on the supply response for a given unchanged relative price of the goods?

(b) If K_X increases, how does the PPF shift? What can you conclude about the effect on the supply of Y? Is the shift horizontally in a larger proportion than the increase in K_X, or smaller? What can you conclude about the effect on the supply response for a given unchanged relative price of the goods?

(c) Again think of the increase in K_X, with an unchanged relative price of the goods. If the supply of X were to increase in the same proportion as the increase in K_X, what must have happened to L_X and L_Y? What must have happened to the marginal products of labor in the two sectors? Is this possible in equilibrium? What can you say now about the supply response of X?

(d)What are the supply effects of an increase in L?

ECO 352 – International Trade – Spring Term 2010
Week 4 Precepts – February 22
EFFECTS OF GROWTH ON A TRADING ECONOMY – SOLUTIONS

Question 1:

The relative demand is

$$\frac{C}{W} = \left(\frac{P_C}{P_W} \right)^{-1/2} = P^{-1/2}, \qquad \text{or} \qquad P = \left(\frac{C}{W} \right)^{-2} .$$

The relative supply is just the endowment ratio. In equilibrium the relative demand must equal the relative supply. So using the endowment ratio for C/W in this equation, we can calculate the equilibrium P.

In the first part, the endowments are 1 and 1. Therefore $P = 1$. France's budget constraint is

$$P_C\, C + P_W\, W = P_W * 1, \quad \text{or} \quad P\, C + W = 1$$

which now becomes simply $C + W = 1$. The relative demand in France is the same as that in this world. Substituting the equilibrium price, we have $C/W = 1$, or $C = W$. Therefore $C = W = 1/2$, and France's utility is

$$U = \frac{\frac{1}{2} * \frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{4} .$$

In the second part, the endowments are 2 for wine and 1 for cheese. Therefore the equilibrium relative price of cheese is

$$P = \left(\frac{1}{2} \right)^{-2} = 4.$$

France's budget constraint is

$$P_C\, C + P_W\, W = P_C * 2, \quad \text{or} \quad P\, C + W = 2$$

which in equilibrium becomes $4\, C + W = 2$. The relative demand is $C/W = 1/2$, or $C = \frac{1}{2}\, W$. Substituting, we have France's consumption quantities $W = 2/3$, $C = 1/3$, and France's utility is

$$U = \frac{\frac{1}{3} * \frac{2}{3}}{\frac{1}{3} + \frac{2}{3}} = \frac{2}{9} .$$

Since $2/9 < 1/4$, the increase in its wine endowment has reduced France's utility!

The economic intuition is that relative demand is very inelastic (the elasticity of the quantity ratio with respect to the price ratio equals $\frac{1}{2}$). Therefore the increased relative quantity of wine reduces the relative price of wine greatly (doubling the relative quantity reduces the relative price by a facto of 4). Since France exports wine, this worsens its terms of trade. This effect is strong enough to outweigh the benefit of the larger endowment.

In trade theory this phenomenon is called *immiserizing growth*. You are invited to think whether this is just a theoretical curiosum, or it has practical relevance.

You are also invited to think about its implications for trade policy. Should France blame the trade for the immiserization, and as a result refuse to trade with Holland? If it did, what would happen to its utility?

Here is the math of the utility maximization. The algebra is simpler if you write the utility function as

$$\frac{1}{U} = \frac{C+W}{C\,W} = \frac{1}{W} + \frac{1}{C}, \quad \text{or} \quad U^{-1} = W^{-1} + C^{-1}\,.$$

Differentiating,

$$-U^{-2}\,\frac{\partial U}{\partial W} = -W^{-2}, \qquad -U^{-2}\,\frac{\partial U}{\partial C} = -C^{-2}\,.$$

Therefore the MRS along an indifference curve is

$$-\left.\frac{dW}{dC}\right|_{U=\text{constant}} = \frac{\partial U/\partial C}{\partial U/\partial W} = \frac{-C^{-2}}{-W^{-2}} = \left(\frac{W}{C}\right)^2\,.$$

Equating this MRS to the relative price of cheese, P_C/P_W, and solving for the quantity ratio, we get the relative demand equation.

Question 2:

The figure of the elliptical PPF is attached. The intercepts X_{max} and Y_{max} on the two axes are found by setting one at a time of X and Y equal to zero in the equation of the ellipse and solving for the other:

$$X_{max} = A_X\ \left(K_X\,L\right)^{1/2}, \qquad Y_{max} = A_Y\ \left(K_Y\,L\right)^{1/2}\,.$$

(a) An increase in A_X stretches the whole ellipse horizontally in the same proportion as the increase in A_X. Suppose the initial tangency with the relative price line was at A as shown in the figure. At the point B on the new ellipse that has the same Y, and X higher equiproportionately with A_X, the tangent must be flatter than the one at A on the old ellipse. Therefore the tangency of the new ellipse with a relative price line of the same slope as at A must occur at a point C to the southeast of B. Thus the supply of X will increase in a greater proportion than the increase in A_X, and the supply of Y will decrease.

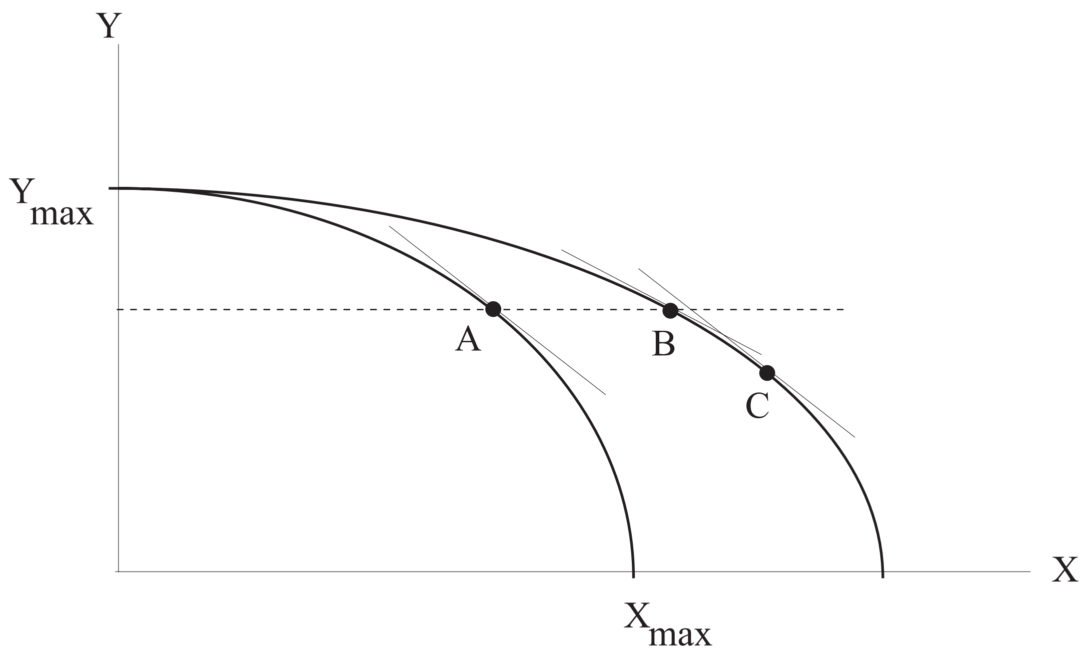
(b) As K_X increases, the ellipse again stretches horizontally, but now in the same proportion as the square root of K_X, and therefore in smaller proportion than the increase in K_X. We can use the same figure to show the stretch but interpret it in the new sense. Once again the tangent to the new ellipse at B is flatter than that to the old ellipse at A. Therefore the new supply response must be at a point like C to the southeast of B. The supply of Y will decrease. The supply of X will increase in greater proportion than the square root of K_X, but we cannot yet say whether it will increase in greater or smaller proportion than the increase in K_X.

(c) If the supply of X were to increase in the same proportion as the increase in K_X, the amount of L_X would also have to increase in the same proportion. This would leave the ratio K_X/L_X unchanged, and therefore the marginal product of labor in the X sector unchanged. But the increase in L_X comes from withdrawing labor from the Y sector. Now in the Y sector less labor works with the same K_Y, so the marginal product of labor there goes up. But this will disturb the equality of the values of marginal products of labor in the two sectors. So the labor shift cannot go that far; the increase in X will be in smaller proportion than the increase in K_X. (Then the K/L ratio will rise equally in both sectors, and the marginal product of labor will also rise equally in both.)

All these derivations are of course specific to the Cobb-Douglas square root production functions. But most of the ideas and the qualitative results – the directions and relative proportions of the supply responses – are perfectly general and can be derived for general production functions using a lot of math.

There is one exception that matters.

(d) In the Cobb-Douglas square root example, an increase in the total labor L enlarges the ellipse radially outward in proportion to the square root of L. The MRT is constant along each ray through the origin. So at the given relative price P_X/P_Y, supplies of both X and Y will increase in the same proportion, namely $\sqrt{L}$. Therefore if two countries differed only in their labor endowments, they would have the same relative supply functions. Neither would have a comparative advantage, and there would be no reason for trade between them (given our maintained assumption of identical homothetic preferences). But more generally, an increase in the total labor endowment can expand the PPF non-radially and comparative advantage can arise due to differences in labor endowments. You can then view the Cobb-Douglas square root case as a borderline case that separates two possibilities, and reminds you that both possibilities exist. The details of how labor endowment differences translate into comparative advantage are more subtle (depend on the wage shares and the elasticities of substitution in production in the two sectors), and are not needed for this course.


Y
Y max
A
B
C
X
X max