ECO 352 – Spring 2010
International Trade
Problem Set 1 – Answer Key

Grade Distribution

90-100	80-89	< 80
23	7	2

General Comments

Overall an excellent start. Please read the answer key carefully to see where you may have lost points, and to improve your understanding of how to reason and write problem set answers more generally. Here are a few common errors:

Q1: Some people lost points for not being specific/detailed in their reasoning for the answers to parts (a) and (b). Some failed to make the connection of "preferring one good more" to MRS. In part (c) a few did not carry through their calculations and merely made comparison of the MRS formulas.

Q2: An alternative way to derive relative price is by equating relative demand (varies with relative price) and relative supply (fixed in endowment economy).

2 points are deducted for using specific illustrative numbers for x and y rather than deriving the full range of x and y explicitly.

Question 1:

The common theme in all the parts is as follows: In any one country, all consumers have identical homothetic preferences, and the endowments are fixed. Therefore the relative price in autarky is the marginal rate of substitution evaluated at the endowment point. Since preferences are homothetic, the MRS depends only on the ratio of the two endowments. When free trade becomes possible, it will actually occur if the two autarkic MRS's differ, and the country with the lower MRS of cheese relative to wine (cheese on the horizontal axis and wine on the vertical axis) will export cheese and import wine.

(a) Here $W_F^*/C_F^* = 3/2 = 6/4 = W_G^*/C_G^*$. The preferences are identical across countries, so the two autarkic MRS's are equal as well. There will be no trade even when all barriers are removed.

(b) Here $W_F^*/C_F^* = 3/1 > 6/4 = W_G^*/C_G^*$. The preferences are identical across countries. So the country with the smaller relative endowment of wine (Guilder) will have a lower relative autarkic price of cheese. When trade opens up, Guilder will export cheese and Florin will export wine. (Each country exports its relatively more abundant good.)

(c) Here $W_F^*/C_F^* = 3/2 = 6/4 = W_G^*/C_G^*$. But the preferences are not identical across countries, so the two autarkic MRS's will differ. There will be trade when it is allowed to occur. Whichever country has the lower relative autarkic price of cheese (here caused by a relatively less taste for cheese) will export cheese.

(d) Here we need to calculate the MRS's explicitly. For Florin,

$$MRS_{C_F \to W_F} = \frac{\partial U_F / \partial C_F}{\partial U_F / \partial W_F} = \frac{W_F}{C_F} = \frac{3}{2} = 1.5 \text{ at the endowment point}\,.$$

For Guilder

$$MRS_{C_G \to W_G} = \frac{\partial U_G / \partial C_G}{\partial U_G / \partial W_G} = \frac{1/2 (C_G)^{-1/2} W_G}{(C_G)^{1/2}} = \frac{W_G}{2\, C_G} = \frac{8}{2 * 4} = 1 \text{ at the endowment point}\,.$$

So Guilder has the lower autarkic MRS and therefore exports cheese when trade opens up.

(Additional remark: Guilder has the larger relative endowment of wine: $W_F^*/C_F^* = 3/2 < 8/4 = W_G^*/C_G^*$. But it has an even stronger relative preference for wine: its consumers want to spend 2/3 of their budget on wine, as opposed to 1/2 in Florin. (How do we know that?) The demand side outweighs the supply side and Guilder ends up importing wine.)

Question 2:

General property throughout this question: the Cobb-Douglas utility function has equal powers for the two goods; therefore each consumer spends half of his income on each good.

(a) TUDOR: The budget constraint of each apple-orchard owner is

$$P_A\ A + P_B\ B = P_A * 1 = P_A\,.$$

Therefore the demand functions are

$$A = \frac{1}{3}\ \frac{P_A}{P_A} = \frac{1}{3}, \qquad B = \frac{2}{3}\ \frac{P_A}{P_B}\,.$$

For each banana-plantation owner, the budget constraint is

$$P_A\ A + P_B\ B = P_B * 1 = P_B\,.$$

Therefore the demand functions are

$$A = \frac{1}{3}\ \frac{P_B}{P_A}, \qquad B = \frac{2}{3}\ \frac{P_B}{P_B} = \frac{2}{3}\,.$$

The equilibrium conditions are: for apples

$$2000\ \frac{1}{3} + 6000\ \frac{1}{3}\ \frac{P_B}{P_A} = 2000$$

yielding $P_B/P_A = 2/3$, and for bananas

$$2000\ \frac{2}{3}\ \frac{P_A}{P_B} + 6000\ \frac{2}{3} = 6000$$

yielding $P_A/P_B = 1.5$. (The two market-clearing conditions yield the same solution for the relative price so one of them is redundant, and only relative prices are determinate; these are standard microeconomic general equilibrium properties you should know from ECO 300 or 310.)

Then the autarkic equilibrium consumption quantities and utilities are

$$\begin{aligned} \text{For each apple-orchard owner:} \quad & A = 1/3,\ B = 1,\ U = 1/3, \\ \text{For each banana-plantation owner:} \quad & A = 2/9,\ B = 2/3,\ U = 8/81. \end{aligned}$$

(b) FORDOR: The budget constraints and demand functions are the same as in part (a). The equilibrium conditions are: for apples

$$3000\ \frac{1}{3} + 4000\ \frac{1}{3}\,\frac{P_B}{P_A} = 3000$$

yielding $P_B/P_A = 3/2$, and for bananas

$$3000\ \frac{2}{3}\,\frac{P_A}{P_B} + 4000\ \frac{2}{3} = 4000$$

yielding $P_A/P_B = 2/3$. (Again the two market-clearing conditions yield the same solution for the relative price, and only relative prices are determinate.)

Then the autarkic equilibrium consumption quantities and utilities are

$$\begin{aligned} \text{For each apple-orchard owner:} \quad & A = 1/3,\ B = 4/9,\ U = 16/243, \\ \text{For each banana-plantation owner:} \quad & A = 1/2,\ B = 2/3,\ U = 2/9. \end{aligned}$$

(c) FREE-TRADING WORLD: The budget constraints and demand functions are the same as in part (a). Now there are 5000 apple-orchard owners and 10000 banana-plantation owners. Therefore the equilibrium conditions are: for apples

$$5000\ \frac{1}{3} + 10000\ \frac{1}{3}\,\frac{P_B}{P_A} = 5000$$

yielding $P_B/P_A = 1$, and for bananas

$$5000\ \frac{2}{3}\,\frac{P_A}{P_B} + 10000\ \frac{2}{3} = 10000$$

yielding $P_A/P_B = 1$. (Again the two market-clearing conditions yield the same solution for the relative price, and only relative prices are determinate.)

Then the autarkic equilibrium consumption quantities and utilities are

$$\begin{aligned} \text{For each apple-orchard owner:} \quad & A = 1/3,\ B = 2/3,\ U = 4/27, \\ \text{For each banana-plantation owner:} \quad & A = 1/3,\ B = 2/3,\ U = 4/27 \end{aligned}$$

(d) In Tudor, apple-orchard owners lose from trade (autarkic utility $1/3 >$ trading utility $4/27$), and banana-plantation owners gain (autarkic utility $8/81 <$ trading utility $4/27$).

In Fordor, apple-orchard owners gain from trade (autarkic utility $16/243 <$ trading utility $4/27$), and banana-plantation owners lose (autarkic utility $2/9 >$ trading utility $4/27$).

(In Tudor under autarky, apples were relatively scarcer and therefore relatively more valuable. Trade brought in Fordor's relatively larger supply of apples. The relative price of apples fell and apple-orchard owners became worse off. The other three comparisons have similar interpretations.)

(e) With the transfers as stated, in Tudor the budget constraint of each apple-orchard owner is

$$P_A \; A + P_B \; B = P_A + 3\, x\, P_B \,.$$

Therefore the demand functions are

$$A = \frac{1}{3} \; \frac{P_A + 3\, x\, P_B}{P_A}, \qquad B = \frac{2}{3} \; \frac{P_A + 3\, x\, P_B}{P_B} \,.$$

For each banana-plantation owner, the budget constraint is

$$P_A \; A + P_B \; B = (1 - x)\, P_B \,.$$

Therefore the demand functions are

$$A = \frac{1}{3} \; \frac{(1-x)\, P_B}{P_A}, \qquad B = \frac{2}{3} \; \frac{(1-x)\, P_B}{P_B} = \frac{2}{3}\, (1-x) \,.$$

In Fordor, the budget constraint of each apple-orchard owner is

$$P_A \; A + P_B \; B = (1 - y)\, P_A \,.$$

Therefore the demand functions are

$$A = \frac{1}{3} \; \frac{(1-y)\, P_A}{P_A} = \frac{1}{3}\, (1-y), \qquad B = \frac{2}{3} \; \frac{(1-y)\, P_A}{P_B} \,.$$

For each banana-plantation owner, the budget constraint is

$$P_A \; A + P_B \; B = 0.75\, y\, P_A + P_B \,.$$

Therefore the demand functions are

$$A = \frac{1}{3} \; \frac{0.75\, y\, P_A + P_B}{P_A}, \qquad B = \frac{2}{3} \; \frac{0.75\, P_A + P_B}{P_B} \,.$$

The equilibrium condition for the (worldwide) apple market is

$$2000\, \frac{1}{3} \; \frac{P_A + 3\, x\, P_B}{P_A} + 6000\, \frac{1}{3} \; \frac{(1-x)\, P_B}{P_A} + 3000\, \frac{1}{3} \; \frac{(1-y)\, P_A}{P_A} + 4000\, \frac{1}{3} \; \frac{0.75\, y\, P_A + P_B}{P_A} = 5000 \,.$$

This simplifies to $P_B/P_A = 1$. As usual, we don't need to consider the banana market equilibrium separately.

(Side-remark: Identical homothetic preferences ensure that the aggregate demands are unaffected by a redistribution of endowments/incomes. With more general preferences, redistribution will change demands and therefore the equilibrium prices. Then proving that transfers can be arranged to benefit everyone is a harder task, but the result remains valid.)

Then the consumption quantities and utilities are:

$$\begin{aligned}
\text{In Tudor for each apple-orchard owner:} \quad & A = \frac{1}{2}\,B = \frac{1}{3}\,(1+3\,x),\ U = \frac{4}{27}\,(1+3\,x)^3, \\
\text{each banana-plantation owner:} \quad & A = \frac{1}{2}\,B = \frac{1}{3}\,(1-x),\ U = \frac{4}{27}\,(1-x)^3, \\
\text{In Fordor for each apple-orchard owner:} \quad & A = \frac{1}{2}\,B = \frac{1}{3}\,(1-y),\ U = \frac{4}{27}\,(1-y)^3, \\
\text{each banana-plantation owner:} \quad & A = \frac{1}{2}\,B = \frac{1}{3}\,(1+0.75\,y),\ U = \frac{4}{27}\,(1+0.75\,y)^3,\ .
\end{aligned}$$

For everyone to gain, we need to ensure

$$\begin{aligned}
\text{In Tudor for each apple-orchard owner:} \quad & \frac{4}{27}\,(1+3\,x)^3 > \frac{1}{3},\ x > \frac{\sqrt[3]{9/4}-1}{3} = 0.103 \\
\text{each banana-plantation owner:} \quad & \frac{4}{27}\,(1-x)^3 > \frac{8}{81},\ x < 1 - \sqrt[3]{2/3} = 0.126, \\
\text{In Fordor for each apple-orchard owner:} \quad & \frac{4}{27}\,(1-y)^3 > \frac{16}{243},\ y < 1 - \sqrt[3]{(4/9)} = 0.237, \\
\text{each banana-plantation owner:} \quad & \frac{4}{27}\,(1+0.75\,y)^3 > \frac{2}{9},\ y > \frac{\sqrt[3]{(3/2)}-1}{0.75} = 0.193\,.
\end{aligned}$$

Thus it is possible to find values of $x \in (0.103, 126)$ and $y \in (0.193, 0.237)$ to ensure that everyone gains from trade. (Additional remark: Note that it is possible to do this using only transfers within each country; no international policy coordination for cross-country transfers is needed.)