

ECO 352 – Spring 2010
International Trade
Problem Set 1 – Due February 18 in class

Question 1: (30 points)

The world of this question consists of two countries, Florin and Guilder, and the economy consists of two goods, Cheese and Wine. The endowments of the two goods in each country are fixed, so this is a pure exchange model. In each country, all consumers have identical homothetic indifference curves; consumers in different countries need not have identical preferences unless specified. The following notation is used:

$$\begin{aligned}C_F^* &= \text{Florin's endowment of Cheese,} \\W_F^* &= \text{Florin's endowment of Wine,} \\C_G^* &= \text{Guilder's endowment of Cheese,} \\W_G^* &= \text{Guilder's endowment of Wine.}\end{aligned}$$

The consumption quantities are denoted by the same symbols without the asterisks, and

$$\begin{aligned}U_F(C_F, W_F) &\text{ is Florin's utility function,} \\U_G(C_G, W_G) &\text{ is Guilder's utility function.}\end{aligned}$$

In each part, the two countries are initially in autarky, and then the possibility of free trade (that is, trade with no trade barriers and no transport costs) opens up. You are asked to find out whether trade will actually occur, and if so, the direction of trade, that is, which country will export which good. You are NOT asked to calculate the volume of trade, or the relative prices after trade, so don't waste your time and effort on that. Each part (a)-(c) is worth 5 points and (d) is worth 15 points.

- (a) $C_F^* = 2$, $W_F^* = 3$, $C_G^* = 4$, $W_G^* = 6$, and the preferences are identical across countries.
- (b) $C_F^* = 1$, $W_F^* = 3$, $C_G^* = 4$, $W_G^* = 6$, and the preferences are identical across countries.
- (c) $C_F^* = 2$, $W_F^* = 3$, $C_G^* = 4$, $W_G^* = 6$, and the preferences are not identical across countries. (Here your answer about the direction of trade will have to be conditional on a precise specification of how preferences differ.)
- (d) $C_F^* = 2$, $W_F^* = 3$, $C_G^* = 4$, $W_G^* = 8$, and the utility functions are

$$U_F(C_F, W_F) = C_F W_F, \quad U_G(C_G, W_G) = (C_G)^{1/2} W_G.$$

Question 2: (70 points)

The world of this question consists of two countries, Tudor and Fordor, and the economy consists of two goods, Apples and Bananas. Each acre-size apple orchard produces a ton of apples, and each acre-size banana plantation produces a ton of bananas, with no other

input. Therefore the endowments of the two goods in each country are fixed, so this is a pure exchange model. All consumers in both countries have identical homothetic preferences given by the Cobb-Douglas utility function

$$U(A, B) = A B^2,$$

where A and B denote the consumption quantities of Apples and Bananas respectively.

Tudor has 8000 consumers, of whom 2000 own an acre-size apple orchard each, and 6000 own an acre-size banana plantation each. Fordor has 7000 consumers, of whom 3000 own an acre-size apple orchard each, and 4000 own an acre-size banana plantation each.

We will consider each country in autarky first, and then let free trade occur. Let P_A and P_B denote the prices per ton of apples and bananas respectively.

(a)(15 points) Start with Tudor in autarky. Write down the budget constraint of each apple-orchard owner; remember that his income comes from his ownership of the apples produced on his orchard. Using the standard results for the Cobb-Douglas utility function, write down his demand functions for the consumption of apples and bananas. Do the same calculation for each banana-plantation owner. Write down the equilibrium conditions for the apple and banana markets in Tudor. Solve these to determine the equilibrium relative price P_A/P_B . Use this to calculate the quantities of apples and bananas consumed, and the utility attained, by each apple-orchard owner and each banana-plantation owner.

(b) (10 points) Repeat all these calculations for Fordor in autarky.

(c) (10 points) Repeat all these calculations for the world with free trade.

(d) (5 points) Who gains, and who loses, from trade?

(e) (30 points) Now suppose there are governments in each country. These use up no resources (!), but implement redistributive policies as follows. In Tudor, each banana-plantation owner gives up a fraction x acre of his ownership right, and the $6000x$ acres so generated are given to the 2000 apple-orchard owners, so each of them now gets income from his ton of apples and $3x$ tons of bananas. In Fordor, each apple-orchard owner gives up a fraction y acre of his ownership right, and the $3000y$ acres so generated are given to the 4000 banana-plantation owners, so each of them now gets income from his ton of bananas and $0.75y$ tons of apples. Write down the budget constraints of the four types of owner-consumers. Write down their demand functions using the standard Cobb-Douglas result; all these will be expressions that are functions of the prices, and of x or y . Calculate the resulting world equilibrium relative price P_A/P_B ; this will not involve x or y . Find expressions for the resulting utilities of the four types of owner-consumers; these will be functions of x or y . By comparing these expressions to the utilities they attained in autarky in parts (a) and (b) above, show that it is possible to find values of x and y so that everyone gains from trade.