ECO 352 – Spring 2010
International Trade
Problem Set 2 – Answer Key

The grade distribution was as follows:

Score/range	100	90-99	80-89	70-79	60-69	50-59
Number of students	14	6	2	3	5	5

General comments:

1. Overall good work, but some low scores mostly for students who missed Q.2. They tend to confuse relative demand and relative supply curves from different models (Leontief preferences versus Cobb-Douglas preferences, pure endowment/exchange model versus Ricardian model, etc.). The RS and RD curves are specific to each model, and you should find and work with the curve appropriate to the model specified in each question.

2. Identical homothetic preferences does not mean equal absolute quantities of consumption for the two countries in trade, only equal relative quantities (because trade equalizes relative prices of goods). Incomes can still differ for the two countries, therefore the scales (absolute quantities) of consumption can differ.

3. Trade does not have to be one-for-one, or in whole numbers. Some students, despite having derived a relative price that was not equal to 1, still insisted on trade being one-for-one.

Question 1: (35 points)

1. (5 points) The amount of labor required to produce one unit of clothing is lower in Sylvania than in Freedonia. Therefore, Sylvania has an absolute advantage in producing clothing as compared to Freedonia. Since the input requirement for producing food is lower in Sylvania as well, Sylvania has an absolute advantage in producing food also

2. (5 points) Consider the ratio of labor requirement in food divided by labor requirement in clothing. For Freedonia this ratio is one, for Sylvania it is 2. If Sylvania reduces its production of food by one unit, it releases two workers who can produce two units of clothing. If Freedonia reduces its production of food by one unit it releases four workers who can produce one extra unit of clothing only. Thus Sylvania is better at producing clothing as compared to producing food relative to Freedonia: Sylvania has a comparative advantage in clothing as compared to food relative to Freedonia. By analogous reasoning, Freedonia has a comparative advantage in producing food as compared to clothing relative to Sylvania.

3. (5 points) The production possibilities frontier (PPF) of each country is a negatively sloped line. The two lines are shown in the top part of Figure 1. Consider first Freedonia. If all 400 man-hours went into clothing, she could produce 10 units of clothing. If the 400 man-hours were expended on food, she could produce, instead,

10 units of food. Connect these two points with a line – the PPF. Do the same for Sylvania, for whom the two points are 40 units of clothing and 20 units of food. The slope of the production possibilities frontier of, say, Freedonia reflects the ratio of the labor coefficients. The country with the comparative advantage in food will have a steeper PPF than will the country with the comparative advantage in clothing.

4. (5 points) The world PPF, shown in the lower part of Figure 1, will have a relatively flat segment and a relatively steep segment. The flat segment will have a slope equal to the slope of the PPF of the country with the comparative advantage in clothing, while the slope along the relatively steep segment equals the slope of the other country's PPF.

5. (5 points) The world consumption point will lie somewhere along a 45 degree line through the origin. This line intersects the world PPF in the part of the frontier that corresponds to Sylvania. This means that Freedonia specializes in food, and Sylvania produces both goods. Since Freedonia produces only food, she exports food and imports clothing. Sylvania, therefore, exports clothing and imports food. The quantities are easily read off from the figure. The total world production (equals consumption) is 20 of each good. Freedonia produces 10 of food; Sylvania produces 10 of food and 20 of clothing. The relative price of clothing equals Sylvania's MRT, namely 1/2. At this price, Sylvania's budget line coincides with its PPF. Since it consumes equal quantities of the two goods, it must consume 13 1/3 units of each. Then Freedonia consumes 6 2/3 of each. Freedonia exports 3 1/3 of food and imports 6 2/3 of clothing.

6. (10 points) The new PPF is shown in Figure 2. Now the consumption line intersects the PPF in the segment that corresponds to Freedonia. Hence, Sylvania specializes in clothing and Freedonia produces both goods. However, the qualitative pattern of trade remains unchanged (because the pattern of comparative ad- vantage has not changed). Freedonia exports food and imports clothing, and Sylvania exports clothing and imports food. The relative price of clothing is now equal to Freedonia's MRT, namely 1. The world production equals consumption is 50 for each good. Sylvania produces 40 of clothing, so Freedonia must produce 10 of clothing and 50 of food. Sylvania consumes 20 of each, exporting 20 of clothing to get 20 of food, while Freedonia consumes 30 of each, exporting 20 of food to import 20 of clothing.

The trading equilibrium in this question can also be found using the relative demand and relative supply approach used in Q.2. Here the relative demand curve will be a vertical line (because of Leontief preferences. We have shown the PPF approach to give you more familiarity with both methods.

Question 2: (65 points)

1. (5 points) Both Home and Foreign can substitute production of wheat for production of corn at a constant rate. The PPFs are therefore straight lines as drawn in Figure 3

below.

2. (10 points) Home's relative supply has three parts. If $p = p_C/p_W < 1$, then home produces only wheat, because this is the production that maximizes income (you can find this point in Figure 3 where the PPF meets the vertical axis – note that in this case, the price line and the PPF are not tangent). If $p = 1$, then Home is indifferent between using its fields to produce corn or wheat: any pattern of production will yield the same value at this relative price. Finally, if $p > 1$, then Home produces only corn. This part of the relative supply curve is not explicitly drawn since the ratio of the two quantities, $r = Q_C/Q_W$, is infinite. Since the utility function is Cobb-Douglas with equal exponents, consumers' optimal expenditures on each good will be equal. Hence, $p_C\ D_C = p_W\ D_W$, and $p = p_C/p_W = D_W/D_C = 1/r$. The graph of this curve is a hyperbola in the (r, p) space. The relative supply and relative demand curves intersect at the point (1,1). Hence, in autarky, $p = 1$ and $Q_C = Q_W$ (=200 each).

   Since the preferences are the same in Foreign, the relative demand curve will be identical to that in Home. The relative supply curve has the same shape as in Home, but the horizontal line is now located at $p^* = 2$ (see Figure 4). The relative demand and supply curves intersect at the point $(\frac{1}{2}, 2)$. Hence, Foreign's autarky price is $p^* = 2$ and $Q^*_C/Q^*W = \frac{1}{2}$ (so $Q^*C = 150$ and $Q^*_W = 300$).

   These relative supply curves do not depend on the economies' endowments (the numbers of fields).

3. (10 points) The World relative demand remains the same, because consumers everywhere share the same preferences. The World relative supply now has five parts. If $p < 1$, then both economies produce only wheat. If $p = 1$, then Home is indifferent between production of either good and Foreign produces only wheat. Home could produce anywhere between $Q_C = 0$ and $Q_C = 400$ units of corn while Foreign produces $Q^*_W = 600$ units of wheat (and $Q^C = 0$). Therefore, the World relative supply, $r = Q^W_C/Q^W_W = (Q_C + Q^*_C)/(Q_W + Q^*_W)$ could be anywhere between 0 and $400/600 = 2/3$ when $p = 1$. If $1 < p < 2$, then Home produces only corn (400 units) and Foreign produces only wheat (600 units). Q^W_C/Q^W_W is then fixed at $400/600 = 2/3$. If $p = 2$, then Home produces only corn and Foreign is indifferent between the production of either good. Home produces $Q_C = 400$ units of corn (and $Q_W = 0$) and Foreign produces anywhere between $Q^*_W = 0$ and $Q^*_W = 600$ units of wheat. Then Q^W_C/Q^W_W can be anywhere between 2/3 (when $Q^*_W = 600$) and infinity (when $Q^*_W = 0$). Finally, if $p > 2$, then both countries produce only corn and there is no world production of wheat, so Q^W_C/Q^W_W must then be infinite (and again, this part of the relative supply can not be explicitly drawn). The relative supply and demand curves, as shown in Figure 5, intersect at the point $(2/3, 3/2)$ on the vertical part of the relative supply curve.

   The world relative supply curve does depend on the sizes of the two economies.

4. (10 points) In this trade equilibrium the two economies are completely specialized.

Home produces only corn ($Q_C = 400$, $Q_W = 0$) and Foreign produces only wheat ($Q^*_C = 0$, $Q^*_W = 600$). We already know the relative demands in both countries, but we do not know the absolute demands (the quantity demanded of each good) since they will depend on income. We must use the budget constraint to obtain these quantities. We know that demand in both countries satisfies $D_W/D_C = p$ or $D_W = p\ D_C$ and the budget constraint $p_C\ D_C + p_W\ D_W = p_C\ Q_C + p_W\ Q_W$, which implies $p\ D_C + D_W = p\ Q_C + Q_W$. Combining these two equations, we obtain:

$$D_C = \frac{1}{2p}\ (p\ Q_C + Q_W) \quad \text{and} \quad D_W = \frac{1}{2}\ (p\ Q_C + Q_W)\,.$$

Replacing with the appropriate values under free trade: the relative price $p = 3/2$, Home's production quantities $Q_C = 400$, $Q_W = 0$ and Foreign's production quantities $Q^*_C = 0$, $Q^*_W = 600$, we obtain the consumption quantities

$$\text{Home: } D_C = 200,\ D_W = 300, \qquad \text{Foreign: } D^*_C = 200,\ D^*_W = 300.$$

(The equality of consumption quantities across countries is a coincidence; equality of the quantity ratios across countries is the consequence of equalization of relative price by trade and identical homothetic preferences.) Home exports $Q_C - D_C = 200$ units of corn in return for $p\ (Q_C - D_C) = 300$ units of wheat.

5. Consequences of change in home productivity.

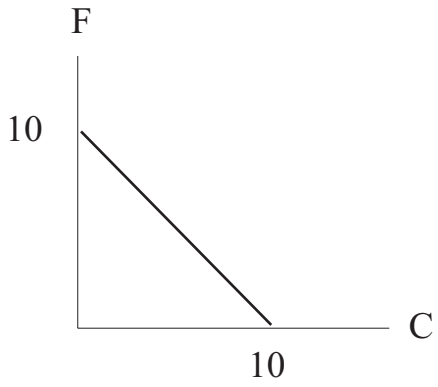
   (a) (10 points) When the productivity of home fields changes to 4, this does not change the relative price $p = 1$ at which home is indifferent between producing corn and wheat. But it does change the maximum quantity of corn home can produce, from 400 to 800. Therefore the horizontal segment of the world relative supply curve at the relative price $p = 1$ is longer. It goes on to the quantity ratio

   $$r = Q^W_C/Q^W_W = (Q_C + Q^*_C)/(Q_W + Q^*_W) = (800 + 0)/(0 + 600) = 4/3\,.$$

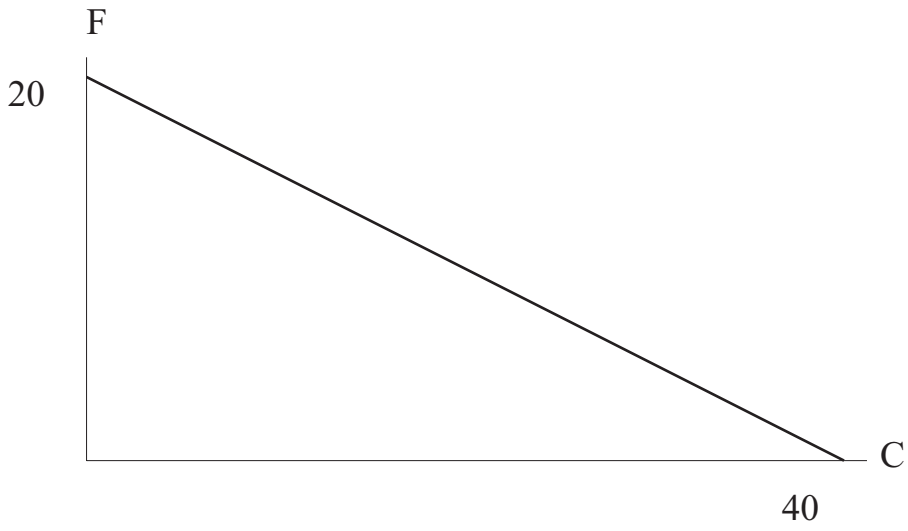
   (b) (10 points) At the relative price $p = 1$, the relative demand ratio is $1 < 4/3$. Therefore the intersection (equilibrium) occurs along the lower horizontal segment ($p = 1$) of the World relative supply curve.

   (c) (10 points) With $p = 1$, Home is indifferent between producing the two crops. Specializing in either will yield 800 units output, and so income 800 in corn units. The budget constraint is $D_C + D_w = 800$. For the given utility function, the income is split equally between the two goods. Therefore $D_C = 400$, $D_W = 400$, and the utility is 160000.

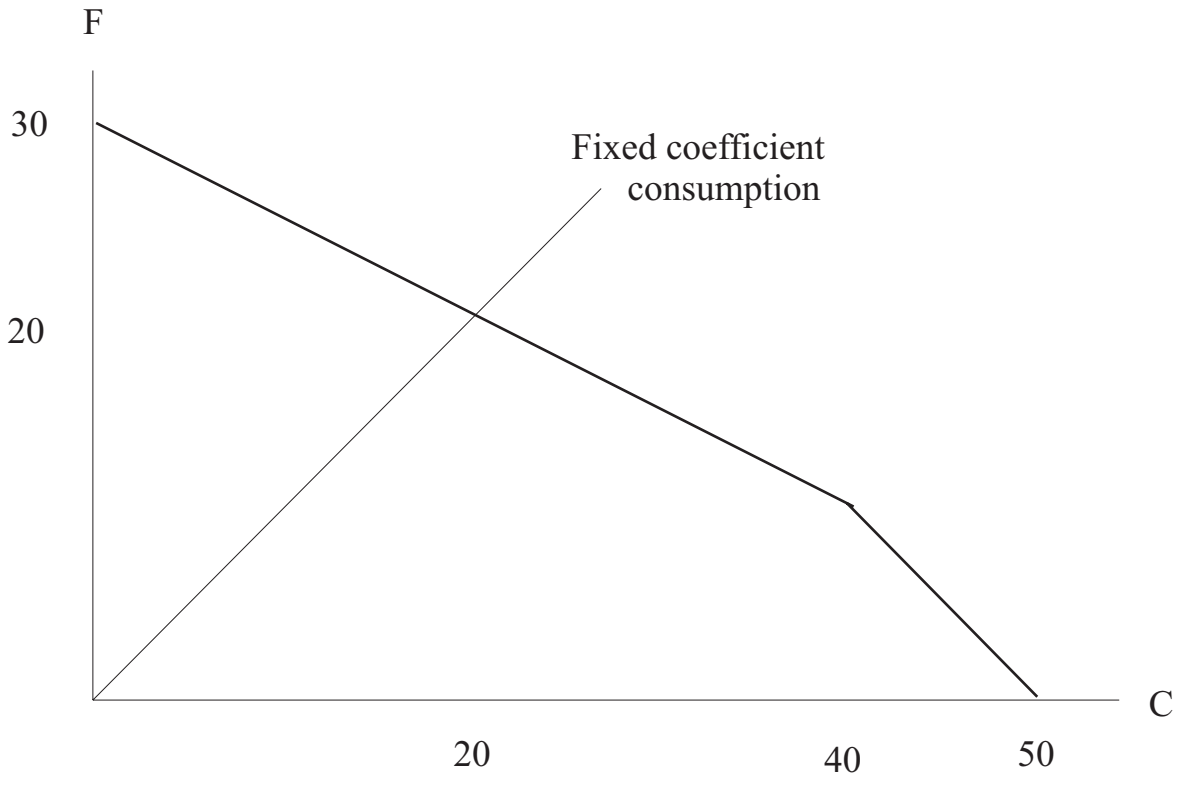
Figure 1



Freedonia

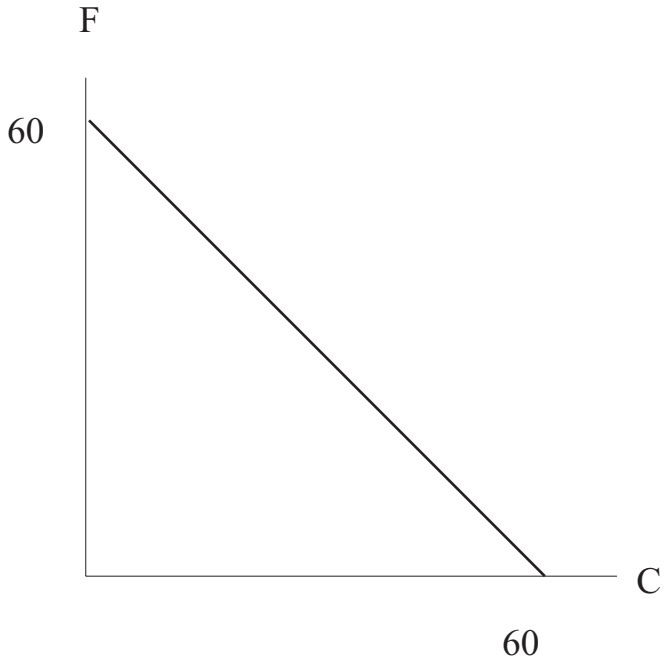


Sylvania

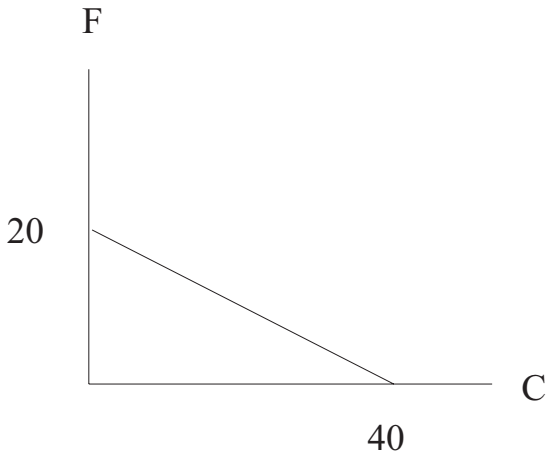


Trading World

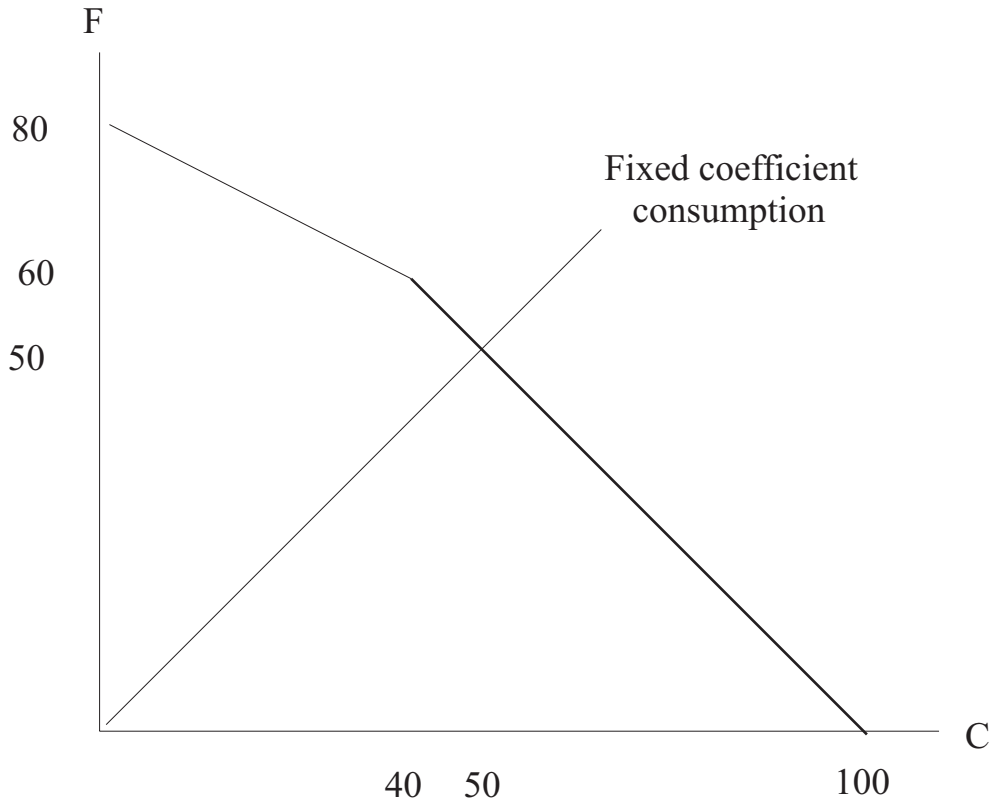
Figure 2



Freedonia



Sylvania



Trading World

Figure 3

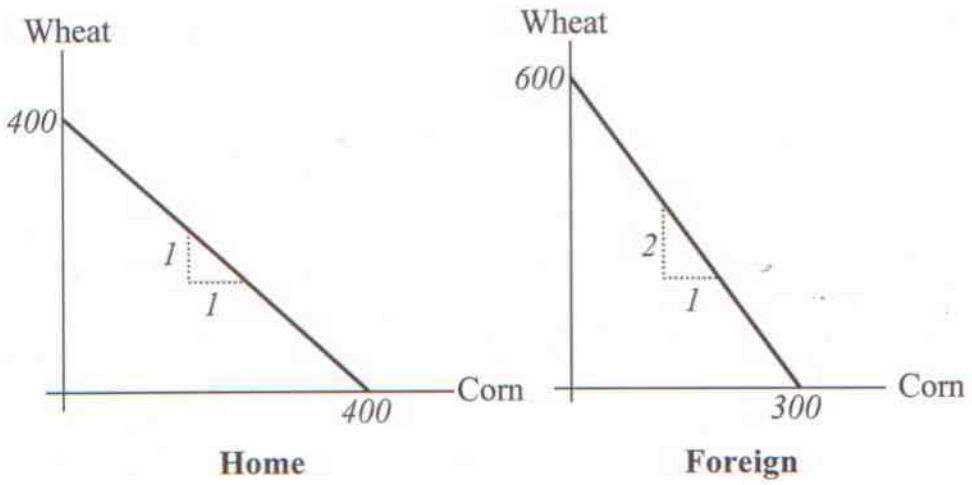


Figure 4

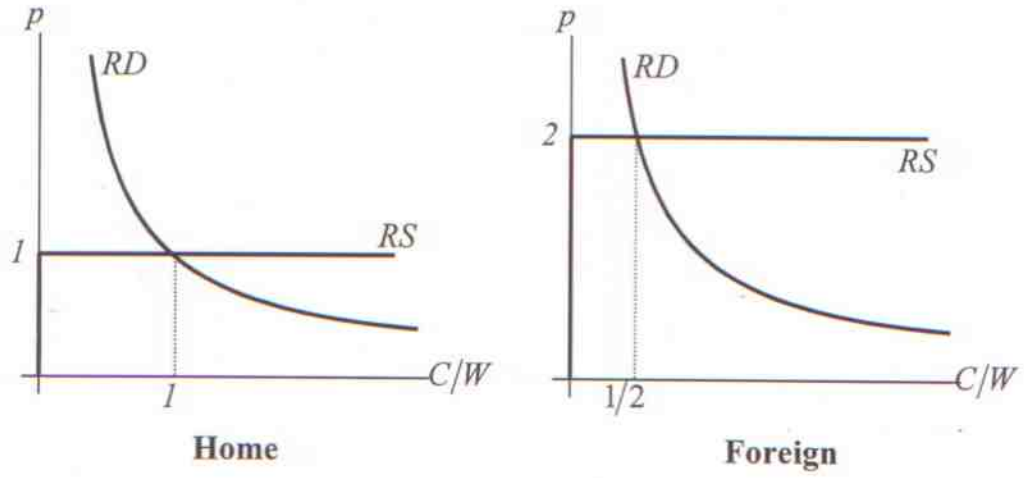


Figure 5