

ECO 352 – Spring 2010
International Trade
Problem Set 3 – Answer Key

Score/range	100	90-99	80-89	70-79	60-69	50-59
Number of students	11	11	3	3	1	1

Overall an excellent performance. One general comment bearing on both questions: It is important to be clear which good is numeraire, both conceptually (what does it mean for a good to be numeraire) and in notation (how relative prices are written and manipulated). More comments will follow in Q.2, especially after part 3(a).

Question 1: (45 points)

(i) Canada's unit labor input requirement ratio Wheat to Autos is $6/27 = 2/9$; that for the US is $8/16 = 1/2$. Therefore Canada has comparative advantage in producing Wheat. We are told that in the trading equilibrium it is producing only one good; that must be Wheat.

More formally, we compare autarkic relative prices. In autarky, each country must produce both goods (so long as there is demand for both). Then in Canada the price equals unit cost (zero pure profit) conditions of equilibrium are:

$$\begin{aligned} P_A(\text{Canada, Autarky}) &= 27 \text{ Wage}(\text{Canada, Autarky}) \\ P_W(\text{Canada, Autarky}) &= 6 \text{ Wage}(\text{Canada, Autarky}), \end{aligned}$$

so

$$\frac{P_A(\text{Canada, Autarky})}{P_W(\text{Canada, Autarky})} = \frac{27}{6} = 4.5.$$

Similarly,

$$\frac{P_A(\text{US, Autarky})}{P_W(\text{US, Autarky})} = \frac{16}{8} = 2.$$

Therefore

$$\frac{P_A(\text{Canada, Autarky})}{P_W(\text{Canada, Autarky})} > \frac{P_A(\text{US, Autarky})}{P_W(\text{US, Autarky})}.$$

Therefore the US has comparative advantage in Autos (and Canada in Wheat).

(ii) Since the US is producing both goods, its relative price $P_A/P_W = 2$ sets the equilibrium price of an Auto relative to (measured in units of) Wheat.

(iii) Do all accounting in units of Wheat (choose Wheat as the numeraire). The prices of autos and wages in the two countries in autarky and trade are:

US (same in trade and autarky):	$P_A = 2,$	$\text{Wage} = 1/8$
(Additional remark: Canada (autarky):	$P_A = 4.5,$	$\text{Wage} = 1/6)$
Canada (trade):	$P_A = 2,$	$\text{Wage} = 1/6$

Therefore the budget constraints are

$$\begin{aligned} \text{US (same in trade and autarky):} & \quad 2 A + W = 1/8 \\ \text{(Additional remark: Canada(autarky):} & \quad 4.5 A + W = 1/6) \\ \text{Canada(trade):} & \quad 2 A + W = 1/6 \end{aligned}$$

For the utility function $U(A, W) = AW$, consumers spend half their income on each good. Therefore the consumption and utility levels for each worker are:

$$\begin{aligned} \text{US (same in trade and autarky):} & \quad A = 1/32, W = 1/16, U = 1/512 \\ \text{(Additional remark: Canada(autarky):} & \quad A = 1/54, W = 1/12, U = 1/648) \\ \text{Canada(trade):} & \quad A = 1/24, W = 1/12, U = 1/288 \end{aligned}$$

(iv) The US comparative advantage in Auto is strengthened. The relative price of Autos, in US autarky as well as in trade, drops to $12/8 = 3/2$. Then the budget constraints with trade become

$$\begin{aligned} \text{US (autarky and trade):} & \quad (3/2) A + W = 1/8 \\ \text{Canada (trade):} & \quad (3/2) A + W = 1/6 \end{aligned}$$

and the consumption and utility levels for each worker:

$$\begin{aligned} \text{US (autarky and trade):} & \quad A = 1/24, W = 1/16, U = 1/384 \\ \text{Canada(trade):} & \quad A = 1/18, W = 1/12, U = 1/216 \end{aligned}$$

As $1/216 > 1/288$, Canadian workers are even better off than in (iii); benefits of US technical progress in Autos are transmitted to Canada.

(v) The US comparative advantage in Autos remains although it is weaker than before: $4/16 < 6/27$. The relative price of Autos, in US autarky as well as in trade, rises to $16/4 = 4$. Then the budget constraints with trade become

$$\begin{aligned} \text{US (autarky and trade):} & \quad 4 A + W = 1/4 \\ \text{Canada (trade):} & \quad 4 A + W = 1/6 \end{aligned}$$

and the consumption and utility levels for each worker:

$$\begin{aligned} \text{US (autarky and trade):} & \quad A = 1/32, W = 1/8, U = 1/256 \\ \text{Canada(trade):} & \quad A = 1/48, W = 1/12, U = 1/576 \end{aligned}$$

As $1/576 < 1/288$, Canadian workers are worse off than in (iii); benefits of US technical progress in Autos are not transmitted to Canada.

(Additional remark: However, note that Canadian workers remain better off trading with the US than they would by going to Autarky, where their utility is only $1/648$.)

(vi) Technical progress in the US Auto industry (which good US exports and Canada imports) benefits Canada, and technical progress in the US Wheat industry (which good the US imports and Canada exports) hurts Canada? The general principle is that if other countries improve their production of the goods your country imports, this benefits your country, and if they become better at producing the good your country exports, that hurts your country.

Additional remark 1: There are two other useful ways to think about this. One is the simple intuition; the former kind of technical progress lets you buy the imports more cheaply, while the latter kind faces your producers with stiffer competition. The other is somewhat more formal, namely terms of trade. Canada's terms of trade are the relative price of its export good, Wheat, relative to its import good, Autos. US technical progress in Autos changes this relative price from $1/2$ to $2/3$; Canada's terms of trade improve. US technical progress in Wheat changes the relative price from $1/2$ to $1/4$; Canada's terms of trade worsen.

Additional remark 2: Paul Samuelson recently wrote an article arguing that productivity improvements in China have hurt the US. Where these improvements have been in industries where the US has been an exporter, the assertion is true, as we see from the above analysis. Many participants in public policy debates on trade jumped on this, saying that a giant of twentieth century economics had at last seen the light and switched against free trade. However, that is a wrong interpretation of what Samuelson says. Note that in the above analysis, even though Canada gains less from trade when the US becomes better at producing Wheat, it continues to gain: utility $1/576 > 1/648$. Switching to protection would make it even worse off. The only way it could go back to the utility level of $1/288$ in (iii) would be to eliminate the US productivity gain. And Samuelson was surely not proposing to do that to China. (How? By destroying their new factories?)

Question 2: (55 points)

General comment: (1) Surprisingly, more than a few students had trouble with the demand functions for a Cobb-Douglas utility function. You should review the subject focusing on the expenditure shares.

The complete diagrams are attached.

1. Here we construct production possibility frontiers and consumption lines.

(a) The full employment conditions in England are

$$\begin{array}{ll} \text{Labor:} & (1/3) G + (2/3) V = 360, \\ \text{Capital:} & (2/3) G + (1/3) V = 600. \end{array}$$

The PPF is shown as the thick segments of the two lines. The consumption proportions line is $V = \frac{1}{4} G$.

(b) The full employment conditions in Italy are

$$\begin{aligned}\text{Labor:} & \quad (1/3) G + (2/3) V = 240, \\ \text{Capital:} & \quad (2/3) G + (1/3) V = 300.\end{aligned}$$

The PPF is shown as the thick segments of the two lines. The consumption proportions line is $V = \frac{1}{4} G$.

2. Here we construct autarkic equilibria.

(a) If England is in autarky, its production potential in Martinis is where the consumption proportions line meets the PPF. This is the point (720,180), so it can produce 180 Martinis. The MRT at this point, and therefore the price of Gin relative to Vermouth in autarky in England, is 1/2.

This point lies on the Labor full employment line, and below the capital full employment line. Therefore in the autarkic equilibrium in England, labor is fully employed but capital is in excess supply and suffers some unemployment.

(Additional remark: Because production of both goods takes place under fixed coefficients, there is no possibility of input substitution along an isoquant to use the excess capital productively, and because preferences are Leontief, there is no possibility of changing the output mix toward the more capital-intensive good to use that capital either. In more general circumstances there would not be unemployment.)

The price equals unit cost conditions are

$$\begin{aligned}\text{Gin:} & \quad P_G = (1/3) W + (2/3) R, \\ \text{Vermouth:} & \quad P_V = (2/3) W + (1/3) R.\end{aligned}$$

Because some Capital is unemployed in the autarkic equilibrium, $R = 0$. English Capital owners obviously can't buy any Martinis. Then $P_G = (1/3) W$ and $P_V = (2/3) W$. Then the price of each Martini is $P_M = 4 P_G + P_V = 2 W$, or $W/P_M = 1/2$. Each worker can buy 1/2 Martini.

(b) If Italy is in autarky, its production potential in Martinis is where the consumption proportions line meets the PPF. This is the point (400,100), so it can produce 100 Martinis. The MRT at this point, and therefore the price of Gin relative to Vermouth in autarky in Italy, is 2.

This point lies on the Capital full employment line, and below the Labor full employment line. Therefore in the autarkic equilibrium in Italy, Capital is fully employed but Labor is in excess supply and suffers some unemployment.

The price equals unit cost equations for Italy are the same as for England because the technology (as defined by the table of input coefficients) is the same:

$$\begin{aligned}\text{Gin:} & \quad P_G = (1/3) W + (2/3) R, \\ \text{Vermouth:} & \quad P_V = (2/3) W + (1/3) R.\end{aligned}$$

In Italy, some Labor is unemployed in the autarkic equilibrium, so $W = 0$. Italian workers obviously can't buy any Martinis. Then $P_G = (2/3)R$ and $P_V = (1/3)R$. Then the price of each Martini is $P_M = 4P_G + P_V = 3R$, or $R/P_M = 1/3$. Each owner of a unit of Capital can buy $1/3$ Martini.

3. Here we examine a trading equilibrium.

- (a) If the relative price of Gin in terms of Vermouth is 1, inspection of graphs of the PPFs shows that the production point in each country will be at the kink point on its PPF, where both factors are fully employed. The output quantities are (840,120) for England and (360,180) for Italy.

At the relative price of 1, the budget constraint for England is

$$G + V = 840 + 120 = 960.$$

With the consumption proportions $G = 4V$, England will wish to consume $(4/5)960 = 768$ units of Gin and $(1/5)960 = 192$ units of Vermouth. Therefore it will wish to export $840 - 768 = 72$ units of Gin and import $192 - 120 = 72$ units of Vermouth.

At the relative price of 1, the budget constraint for Italy is

$$G + V = 360 + 180 = 540.$$

With the consumption proportions $G = 4V$, Italy will wish to consume $(4/5)540 = 432$ units of Gin and $(1/5)540 = 108$ units of Vermouth. Therefore it will wish to import $432 - 360 = 72$ units of Gin and export $180 - 108 = 72$ units of Vermouth.

GENERAL COMMENT: Formally, an equilibrium is a relative price P (or a set of prices) and allocations (production, consumption, etc), such that, for that P :

- 1) every consumer maximizes utility given his/her budget constraint. (In general, consumption is the solution to the budget constraint and the tangency condition $MRS = P$. For Leontief preferences, this is equivalent to $X/Y = \text{fixed ratio}$.)
- 2) firms' production maximizes profit. For the country as a whole, production maximizes the value of GDP on the PPF.
- 3) markets clear (quantity demanded = quantity supplied).

In general, one would set up the market clearance conditions using the optimal quantities demanded and supplied as functions of relative prices, and solve them with the relative prices as unknowns. Here you are given a relative price, and merely asked to verify that everything works out.

Almost all students were able to derive production from 2), some went on to claim that this is an equilibrium because of a) full employment and/or b) quantity supplied satisfies $G/V = 4$. Neither is correct. As we can see, any price between $1/2$ and 2 will make a) and b) (this is by coincidence as it is just the case that two countries full employment output sum up to have $G/V = 4$, but the budget

constraints and therefore the consumption choices will vary for other prices and therefore 3) may not hold.

Most students got consumption from 1) (and income here W and R can be derived from zero profit conditions). Then some claimed that this is an equilibrium because consumption satisfies $G/V = 4$. Of course it must be $G/V = 4$, because this is one of the conditions used to derive consumption. It would hold for the consumer optimal choices for any price.

You must consider both sides (production and consumption) and check equality of quantities demanded and supplied. It suffices to check equality of absolute quantities for any one good, or of relative quantities, because of Walras' Law; this should be stated.

- (b) The price equals unit cost relations are

$$\begin{aligned} \text{Gin:} & \quad P_V = (1/3) W + (2/3) R, \\ \text{Vermouth:} & \quad P_V = (2/3) W + (1/3) R, \end{aligned}$$

where now $P_G = P_V$ since the relative price is 1. These solve to yield $W = P_V$, $R = P_V$ in each country. (Additional remark: This is factor price equalization in action.)

The price of each Martini is $P_M = 4P_G + P_V = 4W + W = 5W$, and similarly $P_M = 5R$. Therefore each worker and the owner of each unit of Capital in each country can buy $1/5$ Martini.

- (c) English capitalists gain from trade ($1/5 > 0$); English workers lose from trade ($1/5 < 1/2$). Italian workers gain from trade ($1/5 > 0$); Italian capitalists lose from trade ($1/5 < 1/3$).

Each country in aggregate gains from trade: England can consume 192 Martinis under trade, as opposed to 180 under autarky; Italy can consume 108 Martinis under trade, as opposed to 100 under autarky.

(Additional remark: Therefore the losers can be compensated out of the gains of the gainers by making suitable transfers.)

Additional remark: Verify that all these results – comparison of relative prices in autarky, their relation to the relative factor abundance and intensity, the pattern of trade, who gains and who loses from trade, . . . – are in conformity with the predictions of the Heckscher-Ohlin model.

Additional remark: In this question the numbers are rigged so that under trade, the world's factor endowments are just right for producing Gin and Vermouth in the desired fixed consumption proportions of 4:1. If that were not the case, even in trade there would be some unemployment. However, in reality there is substitution in both production and consumption, so it is harmless to make these simplifications in a problem set so as to allow simple solutions.

