Note: For the numerical calculations in Q.1, you can either use a calculator and show
the answers to 3 decimal places, or work using paper and pencil and show everything in
fractions. (In exams you will not be allowed to use a calculator, but any calculations you
have to do there will be much simpler.) In Q.2 it will be better to do calculations using
paper and pencil methods.

Question 1: (45 points)
It is sometimes asserted that international trade yields an extra gain by transmitting the
benefits of technical progress from innovating countries to stagnant ones. The following
example is designed to assess the truth of this claim.

Imagine a world with two countries, US and Canada, and two goods, Wheat and Autos.
Labor is the only factor of production; it is mobile across sectors within each country but
not mobile across countries. The following table gives the amounts of Labor needed per unit
output of the two goods in the two countries:

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Autos</td>
<td>16</td>
<td>27</td>
</tr>
</tbody>
</table>

Free trade prevails. Throughout this question, under trade the US produces both goods,
while Canada is completely specialized.

(i) (5 points) Which good does Canada produce? Why?

(ii) (5 points) What is the equilibrium price of an Auto relative to (measured in units
of) Wheat?

(iii) (10 points) Suppose each workers in either country has the utility function \( U(A, W) =
A W \) where \( A \) and \( W \) denote the quantities of Autos and Wheat respectively consumed. How
many Autos, and how much Wheat, will each US worker buy? What is the resulting utility?
Repeat these calculations for each Canadian worker.

(iv) (10 points) Now suppose the amount of Labor required to produce an Auto in the
US drops from 16 to 12, while that required to produce a unit of Wheat stays at 8 as in (iii).
Repeat the calculations in part (iii).

(v) (10 points) Next suppose instead that the amount of Labor required to produce a
unit of Wheat in the US drops from 8 to 4, while the amount required to produce an Auto
stays at 16 as in (iii). Repeat the calculations in part (iii).

(vi) (5 points) Which technical progress benefits Canada, and which one hurts it? What
general principle can you infer?
Question 2: (55 points)

For the purpose of this question, the world consists of two countries, England and Italy, two goods, Gin and Vermouth, and two factors of production, Labor and Capital. All consumers in both countries consume the goods in bundles of fixed proportions (Leontief indifference curves), consisting of four units of Gin and one unit of Vermouth; each such bundle is called a Martini. The factor requirements per unit of output of the two goods are also fixed, and are the same across countries. They are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Gin</th>
<th>Vermouth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>1/3</td>
<td>2/3</td>
</tr>
<tr>
<td>Capital</td>
<td>2/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

The factors are immobile across countries, but mobile across sectors within each country. The factor endowments in the two countries are fixed, and are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>England</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>Capital</td>
<td>600</td>
<td>300</td>
</tr>
</tbody>
</table>

You will be asked to draw various diagrams, and the parts of the question are steps in the construction of parts of the diagrams. You should only submit the final diagrams which show all of the parts of each, labeling all the points and lines appropriately. The diagrams do NOT have to be drawn very accurately to scale; a sketch that shows the relative positions of all the points and the relative slopes of all the lines correctly will suffice.

1. Here we construct production possibility frontiers and consumption lines.

(a) (5 points) Write \( G \) for the output of Gin and \( V \) for the output of Vermouth. Write down the equations that represent the full employment condition for Labor and Capital in England. In a diagram with Gin on the horizontal axis and Vermouth on the vertical axis, show the lines that represent these full employment conditions. Show the production possibility frontier (PPF). Show the line that represents the consumption proportions (the corners of the Leontief indifference curves).

(b) (5 points) Repeat these steps for Italy.
2. Here we construct autarkic equilibria.

(a) (10 points) If England is in autarky, how many Martinis can it produce? Show this point on the PPF. What is the marginal rate of transformation (MRT), and therefore the price of Gin relative to Vermouth in autarky in England? Are both factors fully employed in this situation? If not, which one suffers some unemployment? Write $W$ for the wage rate and $R$ for the return to capital. Write down the equations that represent the zero pure profit (price equals unit cost) conditions for the two goods. Remember that in microeconomics, anything that is in excess supply in equilibrium must have a zero price. Use this to find $W$ and $R$ in England’s autarkic equilibrium. How many Martinis can each English worker and each owner of a unit of capital buy?

(b) (10 points) Repeat these steps for Italy’s autarkic equilibrium.

3. Here we examine a trading equilibrium.

(a) (10 points) Suppose the relative price of Gin in terms of Vermouth is 1. Inspect the PPFs to find the production and consumption choices of the two countries. Is this an equilibrium? Why, or why not?

(b) (10 points) Find the wage and the rate of return to capital in the two countries in the trading equilibrium, again in units of Vermouth.

(c) (5 points) Who gains and who loses from trade?