ECO 352 – Spring 2010
International Trade
Problem Set 5 – Draft Answer Key

The grade distribution was as follows:

Score/range	> 100	100	90-99	80-89	70-79	60-69	$0-59$
Number of students							

Question 1: (75 points)

(a) Expressions for the two companies' profits:

$$\begin{aligned}\Pi_a &= [\,(360 - \tfrac{1}{2}\,Q_a - \tfrac{1}{2}\,Q_b) - C_a\,]\;Q_a - F_a \\ &= (360 - C_a - \tfrac{1}{2}\,Q_b)\;Q_a - \tfrac{1}{2}\;(Q_a)^2 - F_a\,.\end{aligned}$$

Similarly

$$\Pi_a = (360 - C_b - \tfrac{1}{2}\,Q_a)\;Q_b - \tfrac{1}{2}\;(Q_b)^2 - F_b\,.$$

(b) Best response functions: The condition for Airbus to maximize its profit is

$$\partial\Pi_a/\partial Q_a = (360 - C_a - \tfrac{1}{2}\,Q_b) - Q_a = 0\,.$$

This yields the best response function

$$Q_a = 360 - C_a - \tfrac{1}{2}\,Q_b\,.$$

Similarly for Boeing

$$Q_b = 360 - C_b - \tfrac{1}{2}\,Q_a\,.$$

(c) Cournot equilibrium: Solving the two best response functions jointly, we have

$$Q_a = 360 - C_a - \tfrac{1}{2}\;(360 - C_b - \tfrac{1}{2}\,Q_a) = 180 - C_a + \tfrac{1}{2}\,C_b + \tfrac{1}{4}\;Q_a\,,$$

which yields

$$Q_a = 240 - \tfrac{4}{3}\;C_a + \tfrac{2}{3}\;C_b\,.$$

Similarly

$$Q_b = 240 + \tfrac{2}{3}\;C_a - \tfrac{4}{3}\;C_b\,.$$

Then

$$Q = Q_a + Q_b = 480 - \tfrac{2}{3}\;C_a - \tfrac{2}{3}\;C_b\,,$$

$$P = 360 - \tfrac{1}{2}\,(480 - \tfrac{2}{3}\;C_a - \tfrac{2}{3}\;C_b) = 120 + \tfrac{1}{3}\;C_a + \tfrac{1}{3}\;C_b\,,$$

$$\begin{aligned}\Pi_a &= [\,(120 + \tfrac{1}{3}\;C_a + \tfrac{1}{3}\;C_b) - C_a\,]\;Q_a - F_a \\ &= (120 - \tfrac{2}{3}\;C_a + \tfrac{1}{3}\;C_b)\;(240 - \tfrac{4}{3}\;C_a + \tfrac{2}{3}\;C_b) - F_a \\ &= 2\;(120 - \tfrac{2}{3}\;C_a + \tfrac{1}{3}\;C_b)^2 - F_a\,,\end{aligned}$$

and similarly

$$\Pi_b = 2\ (120 + \tfrac{1}{3}\ C_a - \tfrac{2}{3}\ C_b)^2 - F_b\,.$$

(d) No policy; numerical solution: Assume that the quantities are positive and verify this later. Using the numbers $C_a = C_b = 120$ (million dollars), we have

$$Q_a = Q_b = 160,\ Q = 320,\ P = 200,\ \Pi_a = \Pi_b = 7800\,.$$

(Note that since the values are measured in millions of dollars, $F_a = F_b = 5000$.) Since profits are positive, the firms will incur the fixed costs to produce the positive quantities.

For later use, note that with no subsidies, the EU and US welfare levels equal the respective firms' profits.

(e) EU subsidy: Now $C_a = 120 - S_a$, while $C_b = 120$. Substituting in the expressions in part (c) gives,

$$\begin{aligned} Q_a &= 240 - \tfrac{4}{3}\ (120 - S_a) + \tfrac{2}{3}\ 120 = 160 + \tfrac{4}{3}\,S_a\,, \\ Q_b &= 240 + \tfrac{2}{3}\ (120 - S_a) - \tfrac{4}{3}\ 120 = 160 - \tfrac{2}{3}\,S_a\,, \\ Q &= Q_a + Q_b = 320 + \tfrac{2}{3}\ S_a\,, \\ P &= 360 - \tfrac{1}{2}\,(320 + \tfrac{2}{3}\,S_a) = 200 - \tfrac{1}{3}\ S_a\,, \end{aligned}$$

$$\begin{aligned} \Pi_a &= [\,(200 - \tfrac{1}{3}\,S_a) - (120 - S_a)\,]\ Q_a - F_a \\ &= (80 + \tfrac{2}{3}\ S_a)\ (160 + \tfrac{4}{3}\ S_a) - F_a \\ &= 2\ (80 + \tfrac{2}{3}\ S_a)^2 - F_a\,, \end{aligned}$$

and

$$\begin{aligned} \Pi_b &= [\,(200 - \tfrac{1}{3}\,S_a) - 120\,]\ Q_b - F_b \\ &= (80 - \tfrac{1}{3}\ S_a)\ (160 - \tfrac{2}{3}\ S_a) - F_b \\ &= 2\ (80 - \tfrac{1}{3}\ S_a)^2 - F_b\,, \end{aligned}$$

(f) EU's optimal subsidy: The expression for EU welfare is

$$\begin{aligned} W_{EU} &= \Pi_a - S_a\ Q_a \\ &= 2\ (80 + \tfrac{2}{3}\ S_a)^2 - (160 + \tfrac{4}{3}\,S_a)\ S_a - F_a\,. \end{aligned}$$

To maximize this with respect to S_a, the first-order condition is

$$\begin{aligned} 0 = dW_{EU}/dS_a &= 2 \times 2\,(80 + \tfrac{2}{3}\ S_a)\,\tfrac{2}{3} - 160 - \tfrac{4}{3}\,2\,S_a \\ &= \left(\tfrac{640}{3} - 160\right) - \left(\tfrac{8}{3} - \tfrac{16}{9}\right)\ S_a \\ &= \tfrac{160}{3} - \tfrac{8}{9}\ S_a\,. \end{aligned}$$

This yields the solution $S_a = 60$. Then

$$Q_a = 240,\ Q_b = 120,\ Q = 360,\ P = 80,$$

and

$$\Pi_a = 23800,\ \Pi_b = 2200,\ W_{EU} = 9400,\ W_{US} = 2200.$$

Both firms' profits are positive, so the assumption of positive quantities is verified.

Actually this is an incomplete answer. An even higher subsidy will drive Boeing's profit down below zero. When Boeing is no longer in the market, Airbus will enjoy a monopoly.

$$\Pi_b = 2\ (80 - \tfrac{1}{3}\ S_a)^2 - 5000 < 0\,,$$

or

$$80 - \tfrac{1}{3}\ S_a < \sqrt{2500} = 50, \quad \text{or} \quad S_a > 3 \times (80 - 50) = 90.$$

If the subsidy is set just above 90, Airbus as a monopoly facing a demand curve $P = 360 - \frac{1}{2}\,Q_a$ and constant marginal cost $120 - 90 = 30$ will charge price $(360 + 30)/2 = 195$, sell $2 \times (360 - 195) = 330$ planes, and make profit

$$(195 - 30) \times 330 - 5000 = 49450\,.$$

The budgetary cost of the subsidy is $90 \times 330 = 29700$. Therefore EU welfare is $49450 - 29700 = 19750$. This is greater than the 9400 above. So that was only a "local optimum". A few students recognized this and were given bonus points. The rest who did the calculation with duopoly correctly got the full assigned point allocation.

(g) Both governments offer subsidies: $C_a = 120 - S_a$, $C_b = 120 - S_b$. The formulas become

$$\begin{aligned}
Q_a &= 240 - \tfrac{4}{3}\ (120 - S_a) + \tfrac{2}{3}\ (120 - S_b) = 160 + \tfrac{4}{3}\,S_a - \tfrac{2}{3}\,S_b\,,\\
Q_b &= 240 + \tfrac{2}{3}\ (120 - S_a) - \tfrac{4}{3}\ (120 - S_b) = 160 - \tfrac{2}{3}\,S_a + \tfrac{4}{3}\,S_b\,,\\
Q &= Q_a + Q_b = 320 + \tfrac{2}{3}\ S_a + \tfrac{2}{3}\,S_b\,,\\
P &= 360 - \tfrac{1}{2}\,(320 + \tfrac{2}{3}\,S_a + \tfrac{2}{3}\,S_b) = 200 - \tfrac{1}{3}\ S_a - \tfrac{1}{3}\,S_b\,,
\end{aligned}$$

$$\begin{aligned}
\Pi_a &= [\,(200 - \tfrac{1}{3}\,S_a - \tfrac{1}{3}\,S_b) - (120 - S_a)\,]\ Q_a - F_a\\
&= (80 + \tfrac{2}{3}\ S_a - \tfrac{1}{3}\,S_b)\ (160 + \tfrac{4}{3}\ S_a - \tfrac{2}{3}\,S_b) - F_a\\
&= 2\ (80 + \tfrac{2}{3}\ S_a - \tfrac{1}{3}\,S_b)^2 - F_a\,,
\end{aligned}$$

and similarly

$$\Pi_b = 2\ (80 - \tfrac{1}{3}\ S_a + \tfrac{2}{3}\,S_b)^2 - F_b$$

(h) Nash equilibrium of subsidies: Now

$$\begin{aligned}
W_{EU} &= \Pi_a - S_a\ Q_a\\
&= 2\ (80 + \tfrac{2}{3}\ S_a - \tfrac{1}{3}\,S_b)^2 - (160 + \tfrac{4}{3}\,S_a - \tfrac{2}{3}\,S_b)\ S_a - F_a\,.
\end{aligned}$$

To maximize this with respect to S_a taking S_b as given, the first-order condition is

$$\begin{aligned} 0 = dW_{EU}/dS_a &= 2 \times 2\,(80 + \tfrac{2}{3}\,S_a - \tfrac{1}{3}\,S_b)\,\tfrac{4}{3} - (160 - \tfrac{2}{3}\,S_b) - \tfrac{4}{3}\,2\,S_a \\ &= \left(\tfrac{640}{3} - 160\right) - \left(\tfrac{8}{3} - \tfrac{16}{9}\right)\,S_a - \left(\tfrac{8}{9} - \tfrac{2}{3}\right)\,S_b \\ &= \tfrac{160}{3} - \tfrac{8}{9}\,S_a - \tfrac{2}{9}\,S_b\,. \end{aligned}$$

Solving this for S_a gives the EU's best response function to the US subsidy:

$$S_a = 60 - \tfrac{1}{4}\,S_b\,.$$

Similarly the US best response function is

$$S_b = 60 - \tfrac{1}{4}\,S_a\,.$$

Solving the two simultaneously for S_a and S_b gives the Nash equilibrium of subsidies

$$S_a = 48,\ S_b = 48\,,$$

and then

$$Q_a = 192,\ Q_b = 192,\ Q = 384,\ P = 168,$$

and

$$\Pi_a = 13432,\ \Pi_b = 13432,\ W_{EU} = 4216,\ W_{US} = 4216.$$

Once again both firms' profits are positive, so the assumption of positive quantities is valid.

(i) EU subsidy with higher fixed costs: Given the EU subsidy, if there is a solution with firms' Cournot equilibrium with positive quantities, it will still be given by the formulas of part (e), because the fixed costs drop out when we take the first-order conditions with respect to quantities. The optimal EU subsidy will also be 60, following the steps of part (f), for the same reason. The actual numbers for the profits and welfare levels will all be smaller by 3000, however:

$$\Pi_a = 20800,\ \Pi_b = -800,\ W_{EU} = 6400,\ W_{US} = -800.$$

So Boeing has negative profit, and will not produce positive quantity. So the duopoly equilibrium is not tenable in this situation. The equilibrium for a subsidy of 60 must be one where Airbus enjoys a monopoly. It will face the demand curve $P = 360 - \frac{1}{2}\,Q$, and have marginal cost 60. Simple calculation shows that

$$P = 210,\ Q = Q_a = 300,$$

$$\Pi_a = (210 - 60)300 - 8000 = 37000,$$

and

$$W_{EU} = \Pi_a - S_a\,Q_a = 21000.$$

Actually the subsidy of 60 is not needed to keep Boeing out of the market. The smallest subsidy that will do the job is given by

$$\Pi_b = 2\ (80 - \tfrac{1}{3}\ S_a)^2 - 8000 < 0\,,$$

or

$$80 - \tfrac{1}{3}\ S_a < \sqrt{4000} \approx 63.25, \quad \text{or} \quad S_a > 3 \times (80 - 63.25) = 50.25.$$

Again, bonus points were given to the few students who recognized this.

(j) Comments on the findings in parts (d), (f), (h), and (i).

Compare (d) and (f): The EU subsidy increases Airbus's exports and profits, and lowers Boeing's. And EU welfare in (f) is higher than that in (d): 9400 > 7800. The increase in Airbus's profits exceeds the revenue cost of the subsidy. This is because of the strategic advantage the subsidy gives to Airbus in its duopoly with Boeing.

Compare (f) and (h): When both governments join the subsidy game, both end up with lower welfare levels. The subsidies lower the costs for both firms and in the Cournot equilibrium both export more, but the price goes down so that their gains in profits (as compared to the no-subsidy situation) are less than the revenue costs of the subsidy for each government. So the subsidy game is a prisoners' dilemma for the governments. The beneficiaries are consumers in the third country markets who enjoy cheaper air fares.

In (i), the EU gains hugely by being able to keep Boeing out of the market altogether. Here the market is a monopoly and the third country consumers lose as well.

Question 2: (25 points)

The grading of this question is very subjective by its very nature: depends on how many of the relevant points you make, how cogently you make them, and a little on your writing style. In fact we graded this very leniently. The grading of this question is not open to challenge, like balls and strikes.

Although many of the arguments in these articles do not stand up to economic analysis and are based on emotional or political appeal, it is important not to dismiss them out of hand but to marshal sound economic arguments against them.

**** More to come ****