ECO 199 – GAMES OF STRATEGY
Spring Term 2004
FINAL EXAMINATION – MAY 19

READ THESE IMPORTANT INSTRUCTIONS CAREFULLY FIRST:

(1) This is a closed-book exam; put away all your books, manuscripts, notes, notebook or palmtop computers, calculators, cellular phones etc **NOW**. (Paper and pencil methods are sufficient for all the numerical calculations.)

(2) This exam has 4 pages. Make sure you have them all.

(3) We suggest that you spend the first 15 minutes reading the exam carefully and planning your answers. Then plan on approximately 1.5 minutes per point; the points and suggested times are indicated on the first line of each question.

(4) Time will be called after three hours. After that, extra time can be "purchased" at the rate of FIVE points per MINUTE or fraction thereof.

(5) Be brief and write legibly. We prefer you to use a ball-point pen or ink. Writing in pencil may get smudged or unclear; any such unclear answers will be interpreted as wrong. If you change your mind about an answer, make your erasures and corrections VERY CLEARLY and NOT IN PENCIL; any ambiguity will be interpreted as an incorrect answer.

(6) PRINT your FULL NAME on EACH answer booklet. Write out and sign the honor pledge - "I pledge my honor that I have not violated the Honor Code during this examination" - on the cover of your first answer booklet.

QUESTION 1: (10 POINTS, 15 MINUTES)

Give clear and complete but succinct definitions of the following concepts:

[1] Expected payoff
[2] Subgame-perfect equilibrium
[3] Tit-for-tat
[4] Evolutionary stable strategy
[5] Screening

QUESTION 2: (15 POINTS, 30 MINUTES)

Answer the following questions from a game-theoretic perspective. Answer BRIEFLY; for each, plan about 75 words and a tree or a matrix if appropriate.

[a] “If you are playing a two-player sequential-move game where each player moves once, you get a higher payoff if you move first than if you move second.” True or false? Explain.

[b] Dr. Strangelove on the doomsday machine: "It is not only possible for it to be automatically triggered and impossible to de-trigger, it is essential. ... But the whole point of the machine is lost if you keep it a secret." Explain.

[c] When you are selling your car, you wash and polish it. How can this be a signal of quality, since anyone regardless of the quality of the car can mimic it at the same cost?

QUESTION 3: (15 POINTS, 25 MINUTES)

Consider a Chicken game where two players are driving toward each other. Each player chooses between going straight, swerving to the left, and swerving to the right. The choices are made simultaneously. If one goes straight while the other swerves right or left, the one who goes straight gets a payoff of 3 while the other gets –1. If each swerves to his left, each gets 0. If each swerves to his right, again each gets 0. If both go straight, or if one swerves to his left while the other swerves to his right, then the cars crash and each gets the payoff –6.

(a) (5 points) Show the payoff matrix for this game.

(b) (4 points) Find all Nash equilibria in pure strategies.

(c) (6 points) Find the Nash equilibrium in the following kind of mixed strategies: each player chooses to swerve left and swerve right with probabilities x each, and goes straight with probability (1-2x). (This means that you should assume mixtures of this kind, set up all the appropriate equilibrium equations, and use them to solve for x.) Show that for each player the probability of going straight is 6/11.

QUESTION 4: (15 POINTS, 25 MINUTES)

If a sequential-move game is shown in its normal (strategic or matrix) form, we may discover Nash equilibria that do not result from rollback in the tree of the original sequential-move game. Give an example where this happens. What is the significance of the extra Nash equilibria? (5 bonus points if you do not use an example from the textbook or the class overheads, but create a new one of your own for the purpose, of course provided your analysis is correct.)

QUESTION 5: (15 POINTS, 25 MINUTES)

In the 2004 Presidential election, it looks as though George W. Bush will be the Republican candidate and John Kerry will be the Democratic candidate. In addition, Ralph Nader has declared his intention to run as an independent. For simplicity, let's ignore the Electoral College and its

state-by-state winner-take-all rules, and assume that the nation all votes in a single election together. Suppose that American voters have the following preferences (top being best and bottom worst):

Group 1 45% of voters	Group 2 40% of voters	Group 3 15% of voters
Bush	Kerry	Nader
Kerry	Nader	Kerry
Nader	Bush	Bush

(a) (3 points) Suppose the election will be determined by simple plurality. If voters voted truthfully, who would win?.

(b) (9 points) In the plurality election, suppose voters vote strategically. Each group coordinates the votes of all of its members (so all members of one group vote the same way in pursuit of their common preference), but there is no coordination across groups. Find all the pure strategy Nash equilibria of this game. (You can either find the three-dimensional matrix of this game showing the outcomes for all strategy combinations, or try a verbal logical argument, whichever suits you better.)

(c) (3 points) Which of these do you expect as the actual outcome, and why?

QUESTION 6: (30 POINTS, 45 MINUTES)

Here you have a choice between an essay and a mathematical problem. Be aware that a correct solution to the mathematical problem can get you the full 30 points, but errors can drag you down very low. It is very hard to score close to 30 on the essay, and the average score on the essay is lower than the average score on the math problem. But the downside of an essay is generally not so steep either. Also, there are unavoidable subjective elements in the grading of an essay – succinct and well-organized writing counts, not just the correctness of the analysis.

EITHER

Discuss some strategic issues pertaining to bargaining (for example BATNA, patience), asymmetric information (for example product quality, signaling and screening), and strategic moves (for example credibility of promises, reputation) that arise when buying and selling used cars. Hence suggest some good strategies for buyers and for sellers in such deals.

OR (continued on the next page)

Consider a group of 15 players, each of whom has to choose between joining a club (action labeled IN, payoff function labeled J) and staying out (action labeled OUT, payoff function labeled S). The larger the club, the more beneficial it is for any one player to join, and the more costly to stay out. The players are numbered from 1 to 15 in decreasing order of their desire to join. Specifically, the payoffs are as follows:

When n others have chosen IN, for any player k,

if k chooses IN, his payoff is $J(k,n) = n - k$

if k chooses OUT, his payoff is $S(k,n) = 3\ (k - n) - 6$.

Observe that both these payoffs are functions of k and n, where k ranges from 1 to 15, and n ranges from 0 to 14. When n = 0, the choice of "join" by any one player is to be interpreted as founding or forming the club.

(a) (3 points) Show that $J(k,n) > S(k,n)$ if and only if $k < n + 3/2$.

In parts (b) and (c), suppose that the game is played with simultaneous moves.

(b) (3 points) Show that for player 1, joining is the dominant strategy .

(c) (4 points) Show that the game is dominance solvable, and find the equilibrium.

In parts (d) and (e), suppose that the game is played with sequential moves.

(d) (5 points) If the order of moves is 1, 2, 3, ... 15, show that the outcome from the rollback equilibrium is the same as that of the simultaneous-move game above. (Do not try to draw a tree; you can do the rollback reasoning without drawing the game tree.)

(e) (3 points) Give a brief verbal argument to show that the same outcome will result irrespective of the order of moves.

In parts (f)-(h), the equilibrium outcome refers to the common outcome of all the cases (c)-(e) above. The status quo is the situation where the club does not exist at all (n = 0 and everyone has chosen OUT).

(f) (4 points) Which players are better off, and which ones are worse off, in the equilibrium as compared to the status quo?

(g) (4 points) Is the sum of all players' payoffs higher or lower in the equilibrium than in the status quo?

(h) (4 points) Suppose the numerical payoffs are money sums. Then, of the two situations (equilibrium and status quo), whichever has the higher sum of all players' payoffs can in principle be achieved by arranging transfers of money from those who do better to those who do worse. Why might that be difficult to achieve in practice in this instance?
