ECO 199 – GAMES OF STRATEGY
Spring Term 2004
MIDTERM EXAM – Thursday March 11

Important: [1] This is a closed-book exam. No books, manuscripts, notes, calculators, laptop computers, cell phones, ... [2] Write and sign the honor pledge, "I pledge my honor that I have not violated the honor code during this examination" on the front page of your answer book. [3] Time will be called after 50 minutes; extra time can be "bought" at the rate of 4 points of grade per minute. Plan to spend about ½ minute for every point.

Question 1: 30 points

Define each of the following concepts, and in a further couple of sentences for each one, outline their relevance in game theory: (a) Strategy. (b) Nash equilibrium. (c) Screening.

Question 2: 20 points

What are the different roles that mixed strategies play in zero-sum and non-zero-sum games?

Question 3: 50 points

Rhoda, Celeste, and Polly share an apartment. For each, cleaning has a cost 3. If only one of them does the cleaning, the result is worth 2 to each. If two of them do the cleaning, the result is worth 6 to each. If all three work, the result is worth 8 to each. All these costs and benefits are measured in some dollar-equivalents, and costs are subtracted from benefits to arrive at the overall payoffs. For example, if Rhoda cleans the apartment while Celeste and Polly do not, then the payoff to Rhoda is 2 - 3 = -1 and that to each of Celeste and Polly is 2.

Cleaning is scheduled for Saturday afternoon. On Friday evening, when the three are out with separate friends, they make independent decisions of whether to go shopping with these friends on Saturday afternoon, or to stay in the apartment and contribute to the cleaning. Thus their choices are simultaneous and independent.

(a) Show the payoff matrix for the game.

(b) Find all pure strategy Nash equilibria.

(c) Briefly outline, in about 100 words, and without drawing any further trees or matrices, how one or another of the outcomes might get selected.

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The distribution of grades was as follows.

Range	Numbers
100	2
90-99	17
80-89	19
70-79	5
60-69	5
0-59	5

Question 1:

For each, 6 points for definition, 4 points for statement of relevance.

COMMON ERRORS: Generally well done. The most common mistake was in the definition of screening, where people did not mention explicitly that there was a less-informed and a better-informed player (-3). Very few students forgot to mention that a strategy was a "complete" contingent plan (-3). Most of the students lost points because they did not try at all to discuss the relevance of these concepts in game theory (-8 to -12, depending on the quality of the definitions). Only a few students said that strategies need to be complete contingent plans because players are forward-looking (+1 bonus point). Students who discussed information sets also got one bonus point.

(a) Strategy:

Definition: A complete plan of action for a player in a game, specifying the action he would take at all nodes where it is his turn to act according to the rules of the game (whether these nodes are on or off the equilibrium path of play). If two or more nodes are grouped into one information set, then the specified action must be the same at all these nodes.

Relevance: This is the very basic notion of what actions and plans are available to the players in a game. The “complete plan of action” aspect recognizes that players must make plans looking ahead to the actions they would take in various contingencies that can arise.

(b) Nash equilibrium:

Definition: This is a configuration of strategies (one for each player) such that each player’s strategy is best for him, given those of the other players. (Can be in pure or mixed strategies.)

Relevance: This is the central concept of solution or prediction of the outcome of a game. It captures the notion that when a game is played non-cooperatively (each player chooses strategy independently, without any communal enforcement of actions), each will look for all opportunities to do better.

(c) Screening:

Definition: In a game with asymmetric information, this is a strategy of a less-informed player to elicit information credibly from a more-informed player. The typical method is to arrange matters so that the more informed player will have to take an action that will reveal his information credibly.

Relevance: Such strategies are practiced in many contexts in practice, for example when employers look for skilled workers in the applicant pool, insurance companies look for low-risk customers, and airlines try to charge higher prices to travelers who are willing to pay more.

Question 2:

COMMON ERRORS: Most of the students knew that the point of mixing in zero-sum games was to keep the opponent guessing. Whether they discussed maxi-min or not, they got 10 points for that. There was much more confusion in non-zero-sum games, where they discussed mixed vs. pure strategies (even if their discussion applied also to zero-sum games), or non-zero vs. zero sum games (even if it applied to pure strategies as well). Roughly speaking, people who did not mention explicitly the consistency argument got between 10 and 15 (probably 12 on average), and those who did, between 17 and 20. The difference in grades within those two groups comes from the correctness of the additional (and unnecessary) information that they have provided.

In zero-sum games, your mixed strategies serve the role of keeping your opponent guessing, and unable to respond optimally to any systematic pattern of your behavior, because such a response would be to your disadvantage. In technical terms, it raises your maxi-min payoff from the game.

In non-zero-sum games, mixed strategies can be interpreted as subjectively uncertain beliefs about the other player's actions. If your subjective uncertainty about the other player's strategy choice is at just the right level, it can make you objectively uncertain about your own choice, and vice versa. Therefore such uncertainty can be self-supporting in a Nash equilibrium of mixed strategies. However, in a non-zero-sum game there is generally no gain to you from keeping the other player uncertain. Therefore such equilibria are less compelling. They can yield low payoffs to both player; the structure of expectations underlying them is fragile, and they are often unstable.

Question 3:

COMMON ERRORS: Two students were unable to formalize the game (i.e. to get two 2x2 matrices of pay-offs); two other got the Nash Equilibria completely wrong (one of them only looked at dominant strategy and concluded that there was no NE). For the rest of the students, the most common mistake was to ignore the (0,0,0) NE (-5 points per NE missed). In question (c), most students discussed focal points and the possibility to communicate before the game. If everything was right, they got full credit. Those who discussed only one of the two got from 6 to 8 points, depending on the level of details of their answers. Some students also discussed the riskiness of "Shop" (gives both max and min pay-offs), they got 8 points if they did not discuss any other alternative. Some students simply described the thought process of the players that lies behind the Nash equilibrium concept (if I think that the others think that ...), without saying why one rather than another of the many Nash equilibria would be played; they got only two points. For the students who realized that it was probably a repeated game, I gave one bonus point if they discussed how (5,5,5) could be enforced by trigger strategies.

(a) (20 points) The payoff matrix is:

Polly chooses:

Clean

		Celeste	
		Clean	Shop
Rhoda	Clean	5, 5, 5	3, 6, 3
	Shop	6, 3, 3	2, 2, -1

Shop

		Celeste	
		Clean	Shop
Rhoda	Clean	3, 3, 6	-1, 2, 2
	Shop	2, -1, 2	0, 0, 0

In each cell the payoffs are shown in the order (Rhoda, Celeste, Polly).

(b) (20 points) There are four pure strategy Nash equilibria: One where all of them choose to Shop, and three where two choose to Clean and the third chooses to Shop.

(c) (10 points) All three players prefer any one of the three equilibria where cleaning gets done to the equilibrium where all three go shopping. But the inferior equilibrium might prevail because the others involve very unequal distribution of benefits.

The better outcomes that are Nash equilibria of the one-shot game can be sustained (1) in an ongoing or repeated interaction by rotating the cleaning duties, (2) by coordinated randomization.

The jointly best outcome is for them all to clean, yielding total payoff 15. But this is not a Nash equilibrium of the one-shot game. In repeated play that could still be sustained by the threat of responding to any deviation by going to the bad equilibrium (0,0,0).