ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – February 12
SEQUENTIAL-MOVE GAMES (continued)

CRITICISMS OF ROLLBACK THEORY, AND RESPONSES

1. *Too simplistic to do justice to complex trees in real world*

Simple textbook theory is only to give basic understanding
actual use needs building on this to more complex reality
This is no different than most science textbooks
Logic of rollback in simplest games possible without drawing tree
Opposite extreme of complexity – Chess. Full rollback infeasible
People (and computers) use partial rollback
combined with "intermediate valuation function"
A good function of this kind requires "knowledge" and "art"
can combine it with "science" of rollback
Typical application in analysis of economics, politics, business etc.
is intermediate in size and complexity
Needs explicit tree. Solution by inspection, or
use computer programs like Gambit

2. *Rollback not observed even in simple games*
*Classic examples – Waiting in "Centipede" game*
*Sharing in "Ultimatum" and "Dictator" games*

Explanation – Rollback theory is not (necessarily) wrong
theorists often misuse it by assuming wrong payoffs
(1) dividers motivated by fairness – social norm or genetic
(2) choosers dislike unfairness or insult, or dividers fear this
(3) actual game is an ongoing interaction –
exploiting short-run advantage may hurt you later
Such "nicer, fairer" behavior can have long run social value

But inexperienced players can make errors, try experiments
So use equilibrium concept as starting point of analysis

When studying a strategic interaction with sequential moves:

1. Do the rollback analysis
2. See if actual outcomes conform to this
3. If not, double-check the following
   (a) have you have specified the payoffs etc. correctly?
   (b) are the players novices; do they learn by playing?
   (c) is the game too complex?
4. In cases (a) or (b), opportunity for advice / consulting
   With (c), look for heuristics, art

DISTINCTION BETWEEN GAMES WITH
SEQUENTIAL AND SIMULTANEOUS MOVES

Timing is not crucial, what matters is
(1) information when you move
(2) irreversibility of moves

Bidders in a sealed-bid auction may make their choices
and mail their envelopes with bids at different times
Even if A has mailed his bid when B is making his choice,
B does not know the content of A's envelope
So moves are simultaneous

In each play in football, the offensive and defensive coaches
send in their play and alignment simultaneously
But upon observing each other's squad and alignment,
quarterback can change play at line of scrimmage,
defense can realign, either can call a timeout
so moves are closer to sequential

Moves sequential if
(1) later mover can observe earlier moves
(this is one aspect of "perfect information")
(2) the earlier moves cannot be changed secretly
or simultaneously
Otherwise moves are simultaneous

ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – February 12
SIMULTANEOUS-MOVE GAMES

PAYOFF MATRIX (table or spreadsheet) Examples

| "Princess Bride" 1 = live, 0 = dead | | Sicilian | |
|---|---|---|---|
| | | A | B |
| Westley | A | 1 , 0 | 0 , 1 |
| | B | 0 , 1 | 1 , 0 |

Features to note:

(1) Zero-sum
(2) Needs mixing

| Prisoners' Dilemma (smaller numbers better) | | Pianist | |
|---|---|---|---|
| | | Confess | Hold out |
| Tchai-kovsky | Confess | 10 , 10 | 1 , 25 |
| | Hold out | 25 , 1 | 3 , 3 |

Features to note:

(1) One strategy uniformly best
(2) Outcome bad

GENERAL SCHEMATIC "PAYOFF MATRIX"

For two players, Row and Column
Row has available 4 strategies R1, R2, R3, R4
Column has available 3 strategies C1, C2, C3

| | | Column | | |
|---|---|---|---|---|
| | | C1 | C2 | C3 |
| Row | R1 | R11, C11 | R12, C12 | R13, C13 |
| | R2 | R21, C21 | R22, C22 | R23, C23 |
| | R3 | R31, C31 | R32, C32 | R33, C33 |
| | R4 | R41, C41 | R42, C42 | R43, C43 |

SOLUTION CONCEPT – NASH EQUILIBRIUM

a list of strategies, one for each player, such that
each player's strategy is his best response to those of the others,

OR no player can gain from a unilateral deviation

Natural concept of equilibrium for non-cooperative game
In general, games may have no, 1, or many Nash equilibria
So need other concepts or restrictions – later

To illustrate, *suppose* strategy choices (R3,C2)
with payoffs (R32, C32) are a Nash equilibrium.
This requires that

(1) R32 is the largest row payoff in its column
and
(2) C32 is the largest column payoff in its row

We interpret "largest" in weak sense, so only need

R32 $ R12, R32 $ R22, R32 $ R42
and
C32 $ C31, C32 $ C33

ALTERNATIVE INTERPRETATION –
CONSISTENT BELIEFS AND CHOICES

Players move simultaneously,
so neither observes other's choice.
Instead, each forms "belief" (or "theory")
about what the other is choosing.
Each makes his own choice rationally (maximize own payoff)
given his belief about others' choices.
Nash equilibrium is where
all these beliefs happen to be correct