ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – February 17
NASH EQUILIBRIUM: DISCRETE PURE STRATEGIES

SPECIAL CASE OF DOMINANCE

Everything from Row's perspective here; Column similar

Example – Matrix shows Row's payoffs.
For Row, strategy R1 strictly dominates strategy R2
strategy R1 weakly dominates strategy R3
neither of R2, R3 dominates the other

		Column				
		C1	C2	C3	C4	C5
Row	R1	7	5	-1	0	2
	R2	5	4	-7	-3	0
	R3	7	2	-1	-1	1

General definitions –

R1 strictly dominates R2
if for each of column's available strategies,
R1 gives Row a higher payoff than does R2, or
R11 > R21, R12 > R22, R13 > R23, ...
R1 weakly dominates R2 if
R11 $ R21, R12 $ R22, R13 $ R23, ...
with at least one > to avoid trivial cases,
and at least one = to distinguish from strict

(Dominance of one of your strategies over another of your strategies,
NOT dominance of you over another player)

R1 is strictly dominant if it strictly dominates all other strategies;
R1 is strictly dominated if another strategy strictly dominates it;
similar definitions for "weak" versions

If just two strategies are available,
and one is dominated, the other must be dominant
With three or more strategies available,
one may be dominated but none dominant

USE OF THIS CONCEPT

A rational player chooses his dominant strategy (if one exists)
and avoids any dominated strategies
If players know each other to be rational
then their beliefs about others' actions will reflect this
and that in turn will help determine their own best actions
They will also expect others' actions to be similarly affected ...
This narrows down possibilities; can lead to Nash equilibrium

Successively more complex cases
Check that the result in each case is a Nash equilibrium

1. If both sides have dominant strategies
Each will choose his, and believe the other will choose his ...
Example – Prisoners' dilemma (lower numbers better)

		Pianist	
		Confess	Hold out
Tchaikovsky	Confess	10 , 10	1 , 25
	Hold out	25 , 1	3 , 3

2. Only one player has dominant strategy. He will choose it.
The other, with this (correct) belief, will choose his "best response"
Example – (One view of) the US-Soviet nuclear arms race

		U.S.A.	
		Restrained	Aggressive
Soviet Union	Restrained	3 , 4	1 , 3
	Aggressive	4 , 1	2 , 2

3. Dominance solvability – when successive (iterated) elimination of dominated strategies leads to a unique outcome

		Column		
		Left	Middle	Right
Row	Top	3 1	2 3	10 2
	High	4 5	3 0	6 4
	Low	2 2	5 4	12 3
	Bottom	5 6	4 5	9 7

Safe when done using strict dominance;
may miss some equilibria if use weak dominance

		Blue	
		Left	Right
Red	Up	1 , 1	1 , 1
	Down	1 , 1	0 , 0

GENERAL CASES

When no dominance, some other methods for finding Nash eq'a
1. Cell-by-cell inspection – tedious but reliable
2. Best responses and their "intersection"
 Example – Fishermen choose size of catch; Payoffs are profits

		Charlie					
		1	2	3	4	5	6
Ray	1	8, 7	7, 13	6, 15	5, 17	4, 14	3, 9
	2	15, 6	13, 11	11, 13	9, 12	8, 9	5, 3
	3	18, 5	15, 9	13, 11	10, 8	6, 4	3, 4
	4	20, 4	17, 7	12, 8	8, 4	4, 0	0, -2
	5	19, 3	14, 5	9, 3	4, 0	0, -5	-5, -6
	6	15, 2	9, 3	3, 1	-2, -3	-3, -8	-7, -9

3. If zero-sum game, check maximin = minimax
Note – all payoffs are for Row (offense)
Idea - each player makes "most pessimistic" assumption about the other's choice; right thing to do because zero-sum game

		Defense			
		Run	Pass	Blitz	
Offense	Run	2	5	13	min= 2
	Short Pass	6	5.6	10.5	min= 5.6
	Med. Pass	6	4.5	1	min= 1
	Long Pass	10	3	-2	min= -2
		max= 10	max= 5.6	max= 13	

EXAMPLES OF GAMES WITH MULTIPLE NASH EQUILIBRIA

ASSURANCE -

Rousseau's Stag Hunt game

		Barny	
		Stag	Rabbit
Fred	Stag	2 , 2	0 , 1
	Rabbit	1 , 0	1 , 1

Equilibrium selection – focal point (convergence of expectations requires *common knowledge* of choices), communication
(Additional point – Rabbit strategy is "less risky")

BATTLE OF THE SEXES

Choice of movies

		Sally	
		BHD	ABM
Harry	BHD	2 , 1	0 , 0
	ABM	0 , 0	1 , 2

Equilibrium selection – commitment, bargaining, repetition, history

CHICKEN

"Beautiful Blonde" game

		Martin	
		Brunette	Blonde
John	Brunette	3 , 3	2 , 4
	Blonde	4 , 2	1 , 1

Equilibrium selection – commitment, reputation, history/accident

NO NASH EQUILIBRIUM (IN "PURE" STRATEGIES)

Example – soccer penalty kick
Left means goalie’s left etc.
Payoff numbers are kicker’s success probabilities

		Goalie		
		Left	Center	Right
Kicker	Left	0.45	0.90	0.90
	Center	0.85	0	0.85
	Right	0.95	0.95	0.60

Equilibrium in "mixed strategies"

Mixture probabilities calculated using Gambit

Kicker – Left 35.5%, Center 18.8%, Right 45.7%
Goalie – Left 32.5%, Center 11.3%, Right 56.1%

Kicker’s overall probability of success – 75.4%

When each player has a finite number of pure strategies, an equilibrium in mixed strategies always exists. This is Nash’s basic theorem, proved using a "fixed point theorem".