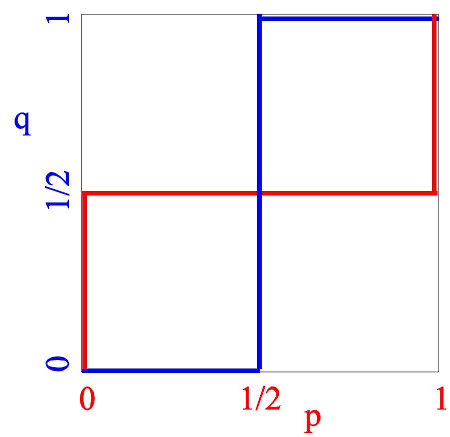
ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – March 2
MIXED STRATEGIES – NON-ZERO-SUM GAMES

ASSURANCE

Stag Hunt example		Barny		
		Stag	Rabbit	q-mix
Fred	Stag	2 , 2	0 , 1	2 q , 2q+(1-q)
	Rabbit	1 , 0	1 , 1	q+(1-q), (1-q)
	p-mix	2p+(1-p), 2 p	(1-p), p+(1-p)	

Barny's best q-response to Fred's p-mix
Stag best ($q = 1$)
if $2p > 1$, or $p > 1/2$
Rabbit best ($q = 0$) if $p < 1/2$
All q equally good if $p = 1/2$

Fred's best p-response to Barny's q-mix
Stag best ($p = 1$)
if $2q > 1$, or $q > 1/2$
Rabbit best ($p = 0$) if $q < 1/2$.
All p equally good if $q = 1/2$



Three Nash equilibria -
Two pure: (1) (p=1, q=1), payoffs (2 , 2) (2) (p=0, q=0), payoffs (1,1)
One mixed: (p=1/2, q=1/2), with expected payoffs
= pq (2,2) + (1-p)(1-q) (1,1) + p(1-q) (0,1) + (1-p)q (1,0)
= 1/4 (2,2) + 1/4 (1,1) + 1/4 (0,1) + 1/4 (1,0) = (1,1)

In mixed-strategy equilibrium, each has correct belief
about the probabilities with which the other will choose actions
"Just right" subjective uncertainty about what the other might do
keeps each objectively unsure about what he himself should do

CHICKEN

"Beautiful Blonde" game		Martin		
		Brunette	Blonde	q-mix
John	Brunette	3 , 3	2 , 4	$3q + 2(1-q)$, $3q + 4(1-q)$
	Blonde	4 , 2	0 , 0	$4q + 0(1-q)$, $2q + 0(1-q)$
	p-mix	$3p + 4(1-p)$, $3p + 2(1-p)$	$2p + 0(1-p)$, $4p + 0(1-p)$	

Martin's best q-response to John's p-mix

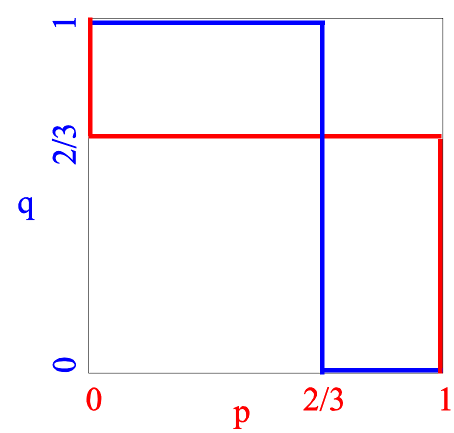
Brunette best ($q = 1$)

if $3p + 2(1-p) > 4p$

$2 - 2p > p$, $p < 2/3$

Blonde best ($q = 0$) if $p > 2/3$

All q equally good if $p = 2/3$



John's best p-response to Martin's q-mix

Brunette best ($p = 1$) if $q < 2/3$

Blonde best ($p = 0$) if $q > 2/3$.

All p equally good if $q = 2/3$

Three Nash equilibria -

Two pure: (1) ($p=0$, $q=1$), payoffs (4, 2), (2) ($p=1$, $q=0$), payoffs (2,4)

One mixed: ($p=2/3$, $q=2/3$), with expected payoffs

$= pq\,(3,3) + p(1-q)\,(2,4) + (1-p)\,q\,(4,2) + (1-p)(1-q)\,(0,0)$

$= 4/9\,(3,3) + 2/9\,(2,4) + 2/9\,(4,2) + 1/9\,(0,0)$

$= (24/9, 24/9) = (2.67, 2.67)$

Worse than (3,3) because of the 1/9 probability of "clash"

Better to do coordinated or correlated randomization

based on some random event both can observe

COMMENTS ON MIXED STRATEGY EQUILIBRIA

1. For most mixtures of other player, your response is pure
    Thus you are willing to mix only for very special mix of other
   That is, probabilities in one player's mix are determined
    by condition of keeping the other indifferent
   Probabilities in your mix change when other's payoffs change,
    not when your own payoffs change !
    (Unless change is big enough to destroy mixed strat. eq'm.)
   This is difficult to grasp for actual players and for students

2. A mixing player is indifferent between all his pure strategies
   Willing to mix, but no positive incentive to choose
    exactly the equilibrium probabilities
   Therefore dynamic stability of the uncertain beliefs is unclear
    if mixed strategy equilibrium is perturbed by some change

3. In zero-sum-games there is genuine reason for mixing –
    other's best response to your pure strategies is worse for you
   (This is why maximin / minimax are relevant in these games)
   And the condition of "keeping the other indifferent"
    is the same as being indifferent yourself

4. In non-zero-sum games, mixed strategy equilibria are sustained
    only by "just-right" subjective uncertainty about others' actions
   Therefore their relevance is more doubtful
    especially because expected payoff can be low
    due to possibility of "clashing" choices
   Will see possible interpretation when doing evolutionary games
   In the assurance game, expect convergence to a pure eqm.
   In Chicken, mixture in population possible

5. Important to choose randomly at each occasion
    People tend to "alternate" too much

6. Mixture probabilities respond to payoff in apparently strange ways:

		Cops	
		City	Suburb
Robbers	City	20	x
	Suburb	80	30

$20\, p + 80\, (1-p) = x\, p + 30\, (1-p), \quad p = 50 / (30+x)$

Example: Old: $x = 70$, $p = 0.500$
New: $x = 90$, $p = 50/120 = 0.417$

As the Robbers' City strategy becomes "better", they use it less.
This seems paradoxical, but only apparently so.

Why? Cops, knowing that the Suburbs strategy is now worse for them, use the City strategy more. So Robbers should use it less.
Not so strange, after all.

Also, equilibrium expected payoff = $(80\, x - 600)/(30+x)$ increases as x increases – "better" strategy is beneficial in payoff sense.