ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – April 8
PRISONERS' DILEMMA

SINGLE PLAY

Each player has two strategies, Cooperate and Defect
Defect is the dominant strategy for each
Both get higher payoffs with (C1,C2) than with (D1,D2)

		Player 2	
		C2	D2
Player 1	C1	C1 , C2	L1 , H2
	D1	H1 , L2	D1 , D2

H1 > C1 > D1 > L1 H2 > C2 > D2 > L2
(Some also require H1 + L1 < 2 C1, H2 + L2 < 2 C2 etc.)

SOLUTION BY REPETITION

General idea – can get extra short-run benefit by defection
but long-run loss because of collapse of cooperation
Need method for comparing payoffs at different points in time
Economics – present discounted values (PDV)
Business – discounted cash flows (DCF)
Logic of compound interest
\$1 today Y \$ (1+r) next year (r = rate of return)
Y \$ (1+r) + r (1+r) = $(1+r)^2$ in two years ...
So \$1 next year = \$ 1/(1+r) today
\$1 in two years = \$ $1/(1+r)^2$ today ...
Today's equivalent PDV of x every year,
starting next year and going on for ever

$$\frac{x}{1+r}+\frac{x}{(1+r)^2}+\frac{x}{(1+r)^3}+\cdots=\frac{x/(1+r)}{1-1/(1+r)}=\frac{x/(1+r)}{r/(1+r)}=\frac{x}{r}$$

Two players can have different rates at which they discount future
Smaller r means future less discounted – player more patient

NUMERICAL EXAMPLE

Two competing ice-cream vendors, Hägen and Dazs.
Each can price High or Low
Profit per unit sold = $3 if High price, $1 if Low price
Each store has 200 loyal customers
There are also floating customers: 400 if best price is High
1400 if best price is Low
If the two stores have unequal prices, floating customers go to lower
If equal prices, they split 50:50

Table of number of customers in 100s

		Dazs	
		High	Low
Hägen	High	4 , 4	2 , 16
	Low	16 , 2	9 , 9

Single-Play payoff table in $100s

		Dazs	
		High	Low
Hägen	High	12 , 12	6 , 16
	Low	16 , 6	9 , 9

FINITE REPETITION

If number of repetitions fixed, finite, and common knowledge
rollback logic Y defection in all rounds
But observation and experiments show
significant cooperation except near the end
Can explain theoretically – based on slight uncertainty about
other person’s behavior, or number of repetitions

INFINITE REPETITION

"Grim Trigger Strategy" –

Complete collapse of tacit cooperation
after a single experience of cheating

High price until one or the other cuts price,
then cut your own price for ever after

One period gain from cheating = 16 - 12 = 4

PDV cost of cheating = (12-9) / r = 3 / r

No cheating if $4 < 3 / r$ or $r < 0.75$ (75 % per year)

"Tit-for-tat" –

Suppose both are playing Tit-for-Tat

Permanent defection has same effect as under grim trigger

Consider deviating for just one period
then suffer low payoff for second period
and get back to cooperation from third period on

Gain 16 - 12 = 4 first year, Lose 12 - 6 = 6 next year

No cheating if $4 < 6 / (1+r)$ or $r < 0.50$ (50% per year)

Generalizations:

Suppose payoffs grow at rate g every period

Probability p that relationship ends in any one period

Condition for deviation to be unprofitable under grim trigger:

$$4 < \frac{3\,(1-p)(1+g)}{1+r} + \frac{3\,(1-p)^2(1+g)^2}{(1+r)^2} + \cdots = 3\,\frac{k}{1-k}$$

with abbreviation $k = (1-p)(1+g)/(1+r)$. This becomes $k > 4/7$. 0.57

If $p = 0.35$, $g = 0.04$, $r = 0.1$, then $k = 0.61$, so barely OK

For other numbers, condition of the form k > some lower limit

Successful cooperation needs:

[1] high g - more likely in growing or stable industries

[2] low p - less likely if fresh entry of outsiders

[3] low r - needs patience, less likely if hit-and-run competitors

OTHERS WAYS OF RESOLVING DILEMMA:

1. Fines or other costs inflicted on cheaters
   Can prevent Defection being dominant strategy
   Can even make Cooperation dominant strategy

2. Promises of rewards for choosing Cooperate
   Can use escrow account for credibility
   May be bilateral, or from larger beneficiary to smaller
   Or from third party

3. Unequal sizes:
   Basic problem of PD is that each player’s defection
      inflicts some cost on the whole group
   If one player is large, enough of this cost comes back
      to him, nullifying his incentive to defect
   Then he may choose to cooperate,
      even knowing that the small fry will defect
   Examples - Saudi Arabia in the OPEC cartel
         US defense expenditures in NATO
         US trade policies in the 1950s to the 70s

EVOLUTIONARY VERSION

Individuals do not rationally choose strategies
Population has different types, each fixed to one strategy
Pairs matched to play PD at random
Strategies with higher payoff increase as % of population
   the less successful ones decrease
In biology, by genetic transmission,
   in social situations, by imitation, learning etc.

Consider an n-fold repetition of our basic PD game;
   payoffs added over the reps, with no discounting

Three types of strategies:

H - always chooses high price (cooperation)

L - always chooses low price (defection)

T - tit-for-tat (choose H on first play, thereafter each time choose what the other chose the previous time)

When T meets L,

L gets 16 the first time and 9 the other (n-1); total 9 n + 7

T gets 6 the first time and 9 the other (n-1); total 9 n - 3

Matrix of payoffs to Player 1

		Player 2 type		
		H	L	T
Player 1 type	H	12 n	6 n	12 n
	L	16 n	9 n	9 n + 7
	T	12 n	9 n - 3	12 n

When n = 2

		Player 2 type		
		H	L	T
Player 1 type	H	24	12	24
	L	32	18	25
	T	24	15	24

So regardless of initial mixture of types in population,

L-types do better than the H and T types

and will eventually become the predominant type

If initially the population is pure T-type

then some H-types can emerge and coexist

But then L-types will emerge and do even better ...

Analogy with dominance under rational play

When n = 10

		Player 2 type		
		H	L	T
Player 1 type	H	120	60	120
	L	160	90	97
	T	120	87	120

Suppose the population is initially all T-type
Some H-types can emerge and coexist
But L-types cannot, so cooperation can be an "equilibrium"
However, if H-types grow to too high a proportion
then an emergent L-type can do better than both of these
Specifically, if proportions x of H-type, (1-x) of T-type, then
expected payoffs to existing H and T types are 120 each
to emergent L-type, $160x + 97(1-x) = 97 + 63x$
Emergent L-type does better if $97 + 63x > 120$, or $x > 23/63$. 0.37
Pure L population is also another equilibrium

Will study more general such "evolutionary games" later

AXELROD'S TOURNAMENTS

Competitors submitted strategy programs
Matched pairwise in "league" format, for 200 repetitions in each pair
Tit-for-tat won first tournament, and won second even though
others knew result of first and honed their strategies against it
General properties that helped TFT:
[1] Nice – never initiates defection
[2] Provocable – retaliates, so never gets beaten too badly
[3] Forgiving – willing to restore cooperation
[4] Simple – opponent can easily figure out what you mean
But if "errors" are possible, Tit-for-Tat gets into long rounds
of retaliatory defection (happened in Axelrod's third tournament)
Can improve by being a little more tolerant