

ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – April 15
COLLECTIVE ACTION GAMES

TWO GENERAL CONCEPTS

1. SPILLOVERS (EXTERNALITIES)

One person's action affects another's payoff

Simplest case: Two players A and B,

each has two choices: inaction (0) and action (1)

Payoffs $A(a,b)$, $B(a,b)$ where a denotes A's choice, b denotes B's

A's action has negative spillover (external effect)

on B if $B(1,b) < B(0,b)$ for $b = 0$ or 1

(in general could depend on what B is doing)

Need not be symmetric,

Similarly with many players.

In many examples, such effects often depend only on the

total numbers taking action, not on individual identities

Examples – (1) traffic congestion, smoking or other pollution

(2) price cut or sales increase by one of competing sellers

Similarly positive spillover or externality

Examples – (1) from beekeepers to fruit growers

(2) getting vaccinated

2. POSITIVE FEEDBACK

A's action increases B's incentive to act also :

$$B(1,1) - B(1,0) > B(0,1) - B(0,0)$$

Example of assurance game:

$$B(1,1) > B(1,0) \text{ but } B(0,1) < B(0,0)$$

Then multiple equilibria can arise

Group can get stuck in worse of these equilibria

Example with many players: Windows vs. Unix ?

CAUSES OF AND REMEDIES FOR INEFFICIENT OUTCOMES

A's choice: action if $A(1,b) - A(0,b) > 0$, inaction if < 0

Ignores addition to group payoff $B(1,b) - B(0,b)$

Therefore we should expect to see

too little of activities that have positive spillovers on others

too much of activities that have negative spillovers on others

To achieve group optimality, must create extra incentive

(+ or – as appropriate) for person conferring spillover

Within the group –

1. Some players do partly “internalize” effects on others

This can reduce inefficiency

2. Enforceable binding contracts with transfer payments

to create the correct incentives for socially optimal actions

3. Repeated interaction, social norms

with detection and enforcement mechanisms

From outside (usually government) – fines, rewards, compulsion

That also create correct incentives

In fact government itself can be thought of as

an overall social contract

Numerous case studies of group attempts to generate

socially optimal outcomes from collective action games:

(Provision of public goods, protection of property rights

controlling depletion of common property resources, ...)

Important features of successful collective action

[1] Stable and small group of players

[2] System designed and monitored by
local people, using better local information

[3] Rules of the game (what is permitted, what sanctions
on violators, ...) clear and well understood

[4] Graduated punishments

The first three conform to general ideas of repeated games

Fourth goes against idea of “grim trigger strategies”

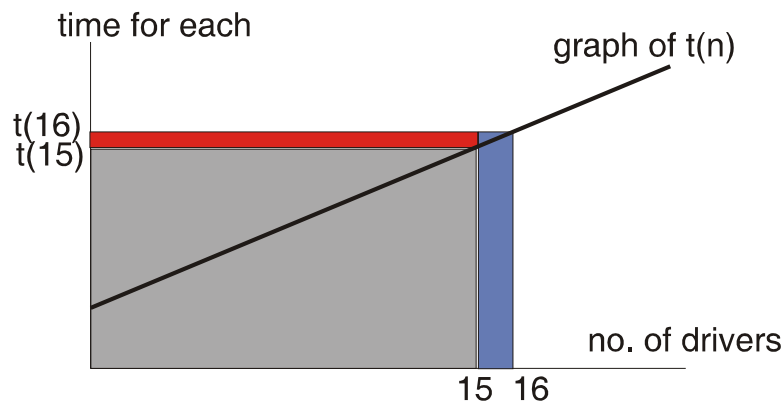
but fits with the “forgiving” aspect of tit-for-tat,

avoids locking in too severe a response to mistakes

AVERAGE AND MARGINAL PAYOFFS

When n drivers on the road, each takes time $t(n)$

Total time $T(n) = n t(n)$



Example – Total time with 15 drivers = gray rectangle

Time taken by 16th driver = blue rectangle

Increase in time for original 15 drivers = red rectangle

Total time with 16 drivers = sum of the three rectangles

If social total net payoff is maximized when 15 drivers, need congestion charge to deter extra 16th driver =

But must charge each because everyone in principle is 16th

General formula: $T(n+1) - T(n) = (n+1) t(n+1) - n t(n)$
 $= t(n+1) + n [t(n+1) - t(n)]$

Marginal cost to group (increase in total time)

= average cost of each member

+ external effect inflicted by one player on all the others

For optimality, charge tax to each player = neg. spillover he creates

Calculus: treating n as continuous variable:

$$TM(n) = d[n t(n)]/dn = t(n) + n t'(n)$$

If $t(n) = a + b n$, $T(n) = a n + b n^2$, and $TM(n) = a + 2 b n$

external effect = $TM(n) - t(n) = b n$

If $t(n) = c n^k$, $T(n) = c n^{(k+1)}$, and $TM(n) = (k+1) c n^k$, so

external effect = $TM(n) - t(n) = k c n^k = k t(n)$

For traffic congestion case, can find $k \cdot 4$

MANY-MALE VERSION OF BEAUTIFUL BLONDE GAME

Active players: n men

Background: 1 blonde, n or more brunettes

Each man chooses Blonde or Brunette

Payoffs:

4 if you are the only man choosing Blonde

3 if you choose Brunette and no one wins the blonde

2 if you choose Brunette and someone else wins the blonde

1 if you choose Blonde and so does at least one other

Asymmetric pure strategy equilibria:

exactly one chooses Blonde, the other (n-1) choose Brunette

Symmetric mixed strategy equilibrium:

each mixes with probabilities p for Blonde, (1-p) for Brunette

Each must be indifferent between the two pure strategies

given the mixed strategies of all the others

Condition for equal expected payoffs:

$$4 * (1-p)^{n-1} + 1 * [1-(1-p)^{n-1}] = 2 * [(n-1) p (1-p)^{n-2}] + 3 * [1 - (n-1) p (1-p)^{n-2}]$$

or

$$(1-p)^{n-2} [3 + n p - 4 p] = 2$$

This can be solved numerically to find p for any given n

Then we can also find probability that

the blonde is not chosen at all, call it $q = (1-p)^n$

Table of solutions:

n	1	2	3	4	5	6	10	20
p	1	0.500	0.268	0.183	0.139	0.113	0.064	0.030
q	0	0.250	0.392	0.446	0.473	0.487	0.516	0.543

As n increases, p goes down so fast that q goes up!