ECO 199 – GAMES OF STRATEGY
Spring Term 2004 – April 20
EVOLUTIONARY GAMES

1. Get away from full rationality
   But allow mechanism that favors better strategies

2. Strategy "genetically" determined
   Each player a "behavioral phenotype"
   Interpretation – rules of thumb, corporate culture, social norm

3. Pairs of players randomly matched
   Variations – (a) whole population plays multi-person game
   (b) individuals from two different species

4. Fitness of a phenotype =
   its expected payoff against random opponent
   Greater fitness implies more offspring. Biol. Definition of fitness
   Our interpretation – imitation, learning, teaching

5. New phenotypes arise by genetic mutation
   Our interpretation – experimentation with new rules of thumb
   A fitter mutant invades successfully

6. Evolutionary stable strategy (ESS)
   Static test – mutant cannot invade population playing ESS
   Dynamic test – from any initial population mix,
   eventually only ESS survives

7. (A) Pure ESS, uniform population (except transient mutants)
   (B) Mixed ESS – (i) Each individual has mixed strategy
   (ii) Mixture in population – "polymorphism"

EXAMPLES

ASSURANCE

Rousseau's Stag Hunt game		Player 2	
		Stag	Rabbit
Player 1	Stag	2 , 2	0 , 1
	Rabbit	1 , 0	1 , 1

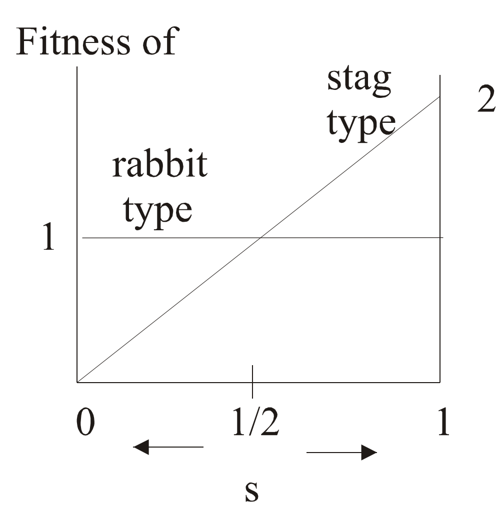
Every day, people are randomly matched in pairs
each gets payoff appropriate from his match
probabilistic average (expectation) over time

Static test shows two pure ESS –
R cannot invade all-S population and vice versa

Suppose population is initially fraction s of stag-type, (1-s) rabbit-type
Fitness of each
stag-type = 2 s + 0 (1-s) = 2 s
rabbit-type = 1 s + 1 (1-s) = 1

Graph these as functions of s
If s > 1/2, s increases further
If s < 1/2, s decreases further
In limit, two pure ESS
So dynamic test gives same result

The ESS correspond to the two
pure strategy Nash equilibria
of the game with rational play



s = ½ is population mixture that corresponds to third (mixed strategy)
Nash equilibrium of the rationally played game, but is not ESS
because it is unstable – destroyed by small deviations

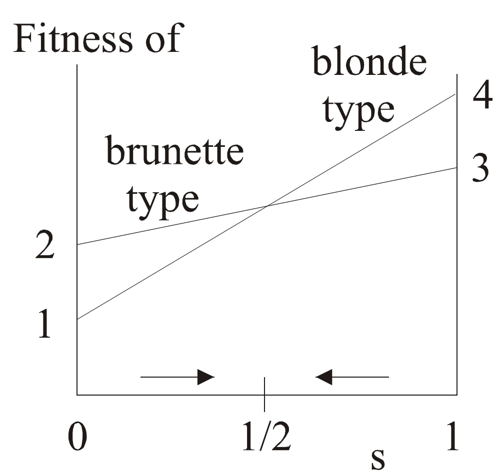
CHICKEN

“Beautiful Blonde” game		Player 2	
		Brunette	Blonde
Player 1	Brunette	3 , 3	2 , 4
	Blonde	4 , 2	1 , 1

Each man randomly matched with another to go to the bar
“Blonde-type” means one who always goes for the blonde, etc.
Success of each depends on who is the partner

Population of pure blonde-types not ESS because
“mutant” brunette-type will get higher payoff (be fitter)
Population of pure brunette-types not ESS becuase
“mutant” blonde-type will get higher payoff (be fitter)

In a population with proportions s brunette-type, (1-s) blonde-type,
Fitness of each
brunette-type = 3 s + 2 (1-s) = 2+s
blonde-type = 4 s + 1 (1-s) = 1 + 3 s



If s < 1/2, brunette-type fitter,
and s increases
If s > 1/2, blonde-type fitter,
and s decreases
In the limit, s = 1/2 :
polymorphic equilibrium

(Imperfect) analogy with the
mixed-strategy Nash equilibrium
of the rationally played game

Also possible in some games to have ESS where
phenotypes (individual strategies) are mixed strategies

REPEATED PRISONERS' DILEMMA

Two-vendor example from April 8
n-fold repetition, but with only two types:
    L (always cheat) and T (tit-for-tat)
Table of Player 1's payoff

		Player 2	
		L	T
Player 1	L	$9n$	$9n + 7$
	T	$9n - 3$	$12n$

Take n to be 3 or higher
Two pure ESS: all-L and all-T
If the population has x type-T and (1-x) type-L, fitness values are
  for L-type = $9n(1-x) + (9n + 7)x$
  for T-type = $(9n - 3)(1-x) + 12nx$
T-type fitter if $(9n - 3)(1-x) + 12nx > 9n(1-x) + (9n + 7)x$
    $(3n - 7)x > 3(1-x)$, $(3n - 4)x > 3$, $x > 3/(3n - 4)$
L-type fitter if $x < 3/(3n - 4)$

So each type is fitter when it is more common in the population
Dynamics converges to one of the two pure ESS
    depending on initial proportion $>$ or $<$ $3/(3n - 4)$
As n gets large, all-T outcome becomes more likely
    Longer-term interaction facilitates emergence of cooperation

But such calculations crucially depend on
    what kinds of mutants can possibly arise
An all-T population can be successfully invaded by
    a mutant S that cheats only in the last play
That in turn by another mutant say S2, which
    cheats on the last two plays, ...
But then T may re-invade if mutant fraction $> 3/(3n - 4)$,
    leading to cycles of population types

GENERAL THEORY

E(I,J) = payoff for I-type when matched against J-type
W(I) = fitness of I-type

Suppose population was all-I,
now a small proportion m of J-mutants arises

W(I) = m E(I,J) + (1-m) E(I,I)
W(J) = m E(J,J) + (1-m) E(J,I)

Mutants cannot invade, and therefore I is ESS, if
Either E(I,I) > E(J,I) primary criterion
Or E(I,I) = E(J,I) and E(I,J) > E(J,J) , secondary criterion

If I is a mixed strategy, made of pure strategies K, L ...
Then necessarily E(I,I) = E(K,I) = E(L,I) ...
so primary criterion is not enough, need secondary

If I is ESS, then it cannot be true that E(I,I) < E(J,I)
So E(I,I) $ E(J,I) (combination of primary and secondary)
If game were rationally played,
I would be best response (at least in weak sense) to itself
So everyone playing I is Nash equilibrium in rational play!

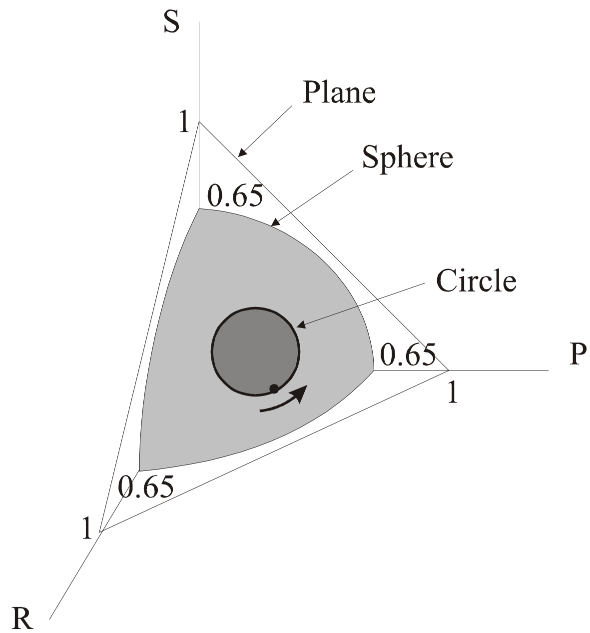
Evolutionary stable implies Nash
Another justification for Nash equilibrium concept

ESS can be used as a criterion for
selecting among multiple Nash eqilibria

MULTI-STRATEGY DYNAMICS

Evolutionary Rock-Paper-Scissors game
Proportions in population R, P, S respectively
Fitness of R = P (-1) + S (1) = S - P
Suppose R increases if this is positive: dR/dt = S - P
Similarly dP/dt = R - S and dS/dt = P - R

Consider $X = R^2 + P^2 + S^2$.
$dX/dt = 2\,R\,dR/dt + 2\,P\,dP/dt + 2\,S\,dS/dt$
$= 2\,[\,R\,(S-P) + P\,(R-S) + S\,(P-R)\,] = 0$
So $R^2 + P^2 + S^2$ = constant, determined by initial conditions
Population proportions cycle along sphere in (R,P,S) space
and of course on plane R + P + S = 1
So point (R,P,S) lies along circle where the
sphere and the plane intersect
(Portion of sphere behind the
plane is shaded light; the part
above the plane is darker.)



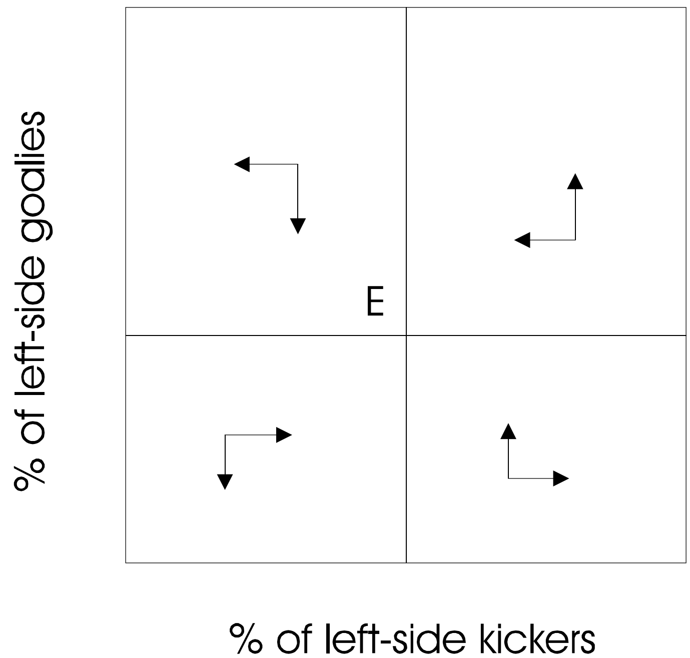
Suppose initially
R = P = 0.4, S = 0.32
Then dR/dt < 0, dP/dt > 0
so point moves along circle
counterclockwise as shown

Book (pp. 454-455) shows
a two-dimensional projection
of this on (R,P) plane

MULTI-SPECIES DYNAMICS

Example – Kickers and Goalies
Each “genetically” left- or right-sided (left = goalie’s left)

Left-sided kickers have higher “fitness” when few left-sided goalies
Left-sided goalies have higher “fitness” when more left-sided kickers



Population proportions of E coincides with probabilities of mixed strategies in eq’m of rational play

Evolutionary dynamics can cycle around E
Let K = proportion of left-side kickers in kickers’ population
G = proportion of left-side goalies in goalies’ population
Suppose mixture probabilities are ½ each, and

$$dK/dt = ½ - G, \quad dG/dt = K - ½$$

Then

$$d [(K-½)^2 + (G-½)^2] = 2 [(K-½) (½-G) + (G-½) (K-½)] = 0$$

so the point (K,G) moves in a circle centered at (½,½).