ECO 199 – GAMES OF STRATEGY
Spring Term 2004
Precepts Week 4

Question 1:

Consider the Beautiful Blonde game of Chicken, with three players – John, Martin, and Charles. There is one blonde and three brunettes, none of them active players in the game. Each of the three players chooses whether to go after the blonde or one of the brunettes. The payoffs to any one of the players are as follows:

5 if this is the only player to choose Blonde
4 if this player chooses Brunette and both of the others choose Blonde
3 if this player chooses Brunette and so do both of the others
2 if this player chooses Brunette and exactly one of the other two chooses Blonde
1 if this player chooses Blonde and so does at least one of the other two

Show the (three-dimensional) matrix of the game, and identify all pure strategy Nash equilibria.

Solution:

Suppose Martin chooses the Page, Charles the Column, and John Row of the matrix The payoff numbers are in order (John, Charles, Martin) .

Page 1, where Martin chooses Blonde, is

		Charles	
		Blonde	Brunette
John	Blonde	1 , 1 , 1	1 , 4 , 1
	Brunette	4 , 1 , 1	2 , 2 , 5

and Page 2, where Martin chooses Brunette, is

		Charles	
		Blonde	Brunette
John	Blonde	1 , 1 , 4	5 , 2 , 2
	Brunette	2 , 5 , 2	3 , 3 , 3

There are three pure strategy Nash equilibria, in each of which one of the three chooses Blonde and the other two choose Brunette. These outcomes are shaded in the matrix above.

Question 2:

You are the Column player playing the following game

		Column	
		Left	Right
Row	Up	1 , 0	1 , 1
	Down	0 , 0	0 , – 9999

Do you choose Left or Right? Why?
What is the Nash equilibrium?
If you made a choice that is not Nash equilibrium, can you reconcile the apparent paradox?

Discussion:

This is like the game on p.141 of the textbook. The game is dominance solvable - Up is Row's dominant strategy, so Column should choose Right. This is the only Nash equilibrium.

But it is risky - if Row has misunderstood the game, or his hand trembles when he is making the choice, then Right may get Column a very bad payoff. So many Column players might choose the safer Left.

This can be justified as a Nash equilibrium of a richer game where the possibility of Row making an error or misunderstanding the payoffs is explicitly incorporated. If p is the probability that Row makes such an error and plays Down, then Column prefers to play Left if

$$0 * (1-p) + 0 * p > 1 * (1-p) - 9999 * p, \quad \text{that is, } 0 > 1 - 10000\,p, \text{ or } p > 1/10000.$$

Question 3:

These are all pure coordination games with individual (non-cooperative) play and no communication. The ability to achieve the mutually preferred outcome depends on the ability to find a focal point or convergence of expectations, often based on a commonly understood "principle".

[1] Two students independently choose any one of the following set of letters and numbers: {0, 1, 7, 13, 100, A}. If the two make the same choice, each gets $1. If they make different choices, neither gets anything.

(Various possibilities: smallest number, largest number, lucky number, unlucky number, the letter because it is unique ...)

[2] Two students independently choose from three colors: Orange, Crimson, and Blue. The payoffs are as follows:

		Student 1					
		Orange		Crimson		Green	
Student 2	Orange	1	1	0	0	0	0
	Crimson	0	0	2.50	2.50	0	0
	Green	0	0	0	0	2.50	2.50

(Princeton students may find Orange focal but that has a lower payoff. Might they go for the middle one? Mix between Crimson and Blue? ...)

[3] Each of two students is to divide the following list of eleven cities into two subsets of any sizes (can go from 10-1 to 6-5). If they choose identical divisions, each gets $2.50. If different, neither gets anything.

Atlanta, Boston, Chicago, Dallas, Denver, Houston, Los Angeles, New York, Philadelphia, San Francisco, Seattle.

(Possible principles - Alphabetical? Size? But then where to make the cut? Usually a geographic principle works best - East v. West of Mississippi.)