

ECO 199 – GAMES OF STRATEGY
Spring Term 2004
Precepts Week 7 – March 22-23 Questions
GAMES WITH ASYMMETRIC INFORMATION – QUESTIONS

Question 1:

In the final stages of the printing of *Games of Strategy*, Sue Skeath and I had to correct the page proofs for about 600 pages. (For those of you who don't know what this entails, it means reading in parallel the manuscript and the typesetters' rendering of it, and mark the errors that the typesetters had committed, so they can correct the final files for printing.) We were both very busy with preparation of lecture, precept, and problem set materials for our Fall term courses; therefore we had to hire a student to do the proofreading.

When it came to devising a contract of work and payment for this job, we had to recognize some problems of information asymmetry. What are they? What payment schemes might cope with them?

Question 2:

Do *Games of Strategy*, Chapter 9, Exercise 8, pp. 298-9.

Use this solution to discuss the general idea of bluffing in poker.

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Question 1:

We want the student to catch all the typesetting errors there may be, but we cannot be sure of that unless we do the checking ourselves, which defeats the whole purpose of hiring the student. The student's effort is unobservable - she takes away the materials and comes back a week later to tell us what errors she has found. What is worse, even the outcome cannot be observed immediately. We will find out about any errors she failed to catch only when some other reader tells us about them. Therefore if the student is paid a flat sum or salary for the job, she has the temptation to shirk -- just hold on to the materials for a few days and then tell us that there are no errors. But if we offer a piece rate (so much per error she finds), she may worry that the typesetter may have done a perfect job, in which case she will have to spend a week or more on the work and get no money at the end of it. She will be reluctant to take the job on these terms.

Therefore the compensation scheme has to be a compromise between the two extremes -- a flat sum plus a bonus per error she discovers. The two components should be so balanced as to give her enough assurance of the total compensation to make the job attractive enough, and enough incentive to attempt a thorough reading.

Our solution was a dollar per page (\$600 total) as a flat sum, plus \$1 per error found (there were 274). I don't claim it was fully optimal or the best deal we could have got. I am waiting to see how many errors remain. So far only seven have come to light.

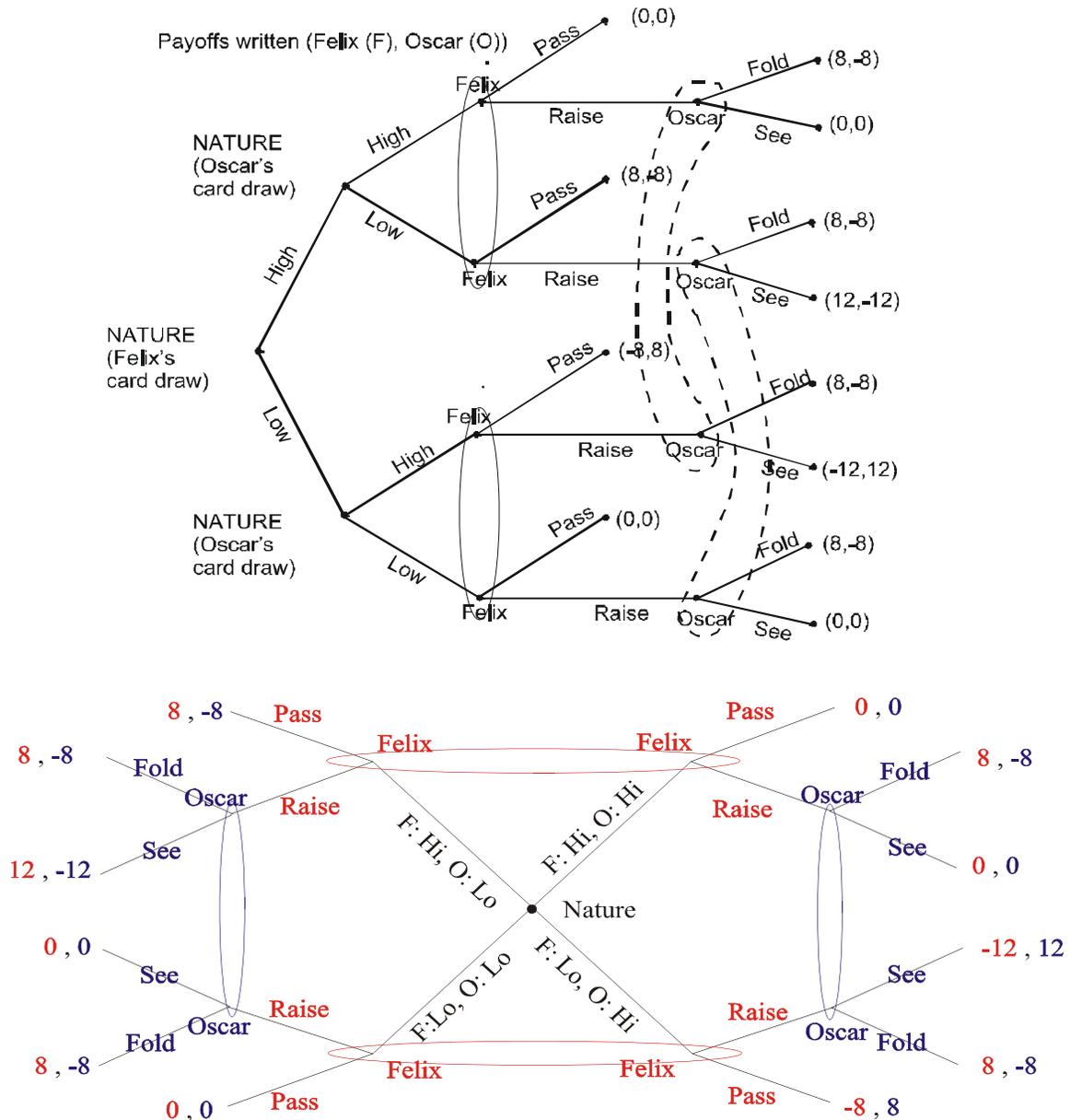
Observe that the incentive scheme focuses on moral hazard. There may be adverse selection, namely the quality of the student. In this situation that is relatively easy to deal with, because we can choose a student who has taken courses from one of us so we have quite good information about her. There are also other points that are relevant in this context: [1] The student may be motivated by considerations other than the immediate monetary sum, e.g. the possibility of a research assistantship later, or a good letter or recommendation when applying for a job or to graduate school. [2] The student cannot "manufacture" errors. If the proofreading were done by the typesetting firm itself, then rewarding them for catching errors at the proofreading stage will have perverse effect on their incentives to do a good job of typesetting in the first place.

Some other neat ideas came up in precept discussions. Here are a couple, with some reasons why we had decided against them.: [1] Hire two students. They compete; only the one who finds more errors gets paid. But to induce students to take on the job under these terms, we would have to offer a very large prize. Also, the students might collude. [2] The student gets a flat sum minus deduction based on an estimate of missed errors. We are to make this estimate by sampling a few pages. The problem with this scheme is that with about 1 error every 2 pages, we would have to sample a lot to get a sufficiently good estimate. Also, the purpose of the scheme is to make her do a careful job, and if that is achieved, we don't need to do any actual checking. In other words, our threat of the deductions is not credible. So it may not work. [3] There may be adverse selection on the other side: the student may think that the book must be especially difficult or error-ridden if the professor is hiring someone else to do the job.

Question 2:

(a) There are two ways in which the game tree can be shown. If you insist on the tree going from left to right, the information sets are quite messy. If you are willing to accept a tree that grows outward from a central initial node, the information sets can be shown more neatly. Both are shown on page 2.

(b) The table on page 3 shows how the expected payoffs are computed. For an example of how the entries are computed, consider that if Felix uses strategy RP, and Oscar uses SF, and the cards drawn are: (i) HiHi, then Felix raises, Oscar sees, and Felix gets 0; (ii) HiLo, then Felix raises, Oscar folds, and Felix gets 8; (iii) LoHi, then Felix passes and gets -8; (iv) LoLo, then Felix passes and gets 0. Each of these four outcomes is equally likely, so the expected payoff is $(1/4)(0 + 8 - 8 + 0) = 0$.



Felix's play after:		Oscar's play after:		Felix's payoff when card draws are:				Felix's expected payoff
Hi	Lo	Hi	Lo	HiHi	HiLo	LoHi	LoLo	
P	P	F	F	0	8	-8	0	0
P	P	S	S	0	8	-8	0	0
P	P	S	F	0	8	-8	0	0
P	P	F	S	0	8	-8	0	0
R	R	F	F	8	8	8	8	8
R	R	S	S	0	12	-12	0	0
R	R	S	F	0	8	-12	8	1
R	R	F	S	8	12	8	0	7
R	P	F	F	8	8	-8	0	2
R	P	S	S	0	12	-8	0	1
R	P	S	F	0	8	-8	0	0
R	P	F	S	8	12	-8	0	3
P	R	F	F	0	8	8	8	6
P	R	S	S	0	8	-12	0	-1
P	R	S	F	0	8	-12	8	1
P	B	F	S	0	8	8	0	4

(c) Felix's strategies PP and PR are weakly dominated. Oscar's strategies FF and FS are also weakly dominated. If we eliminate these, the resulting payoff table is:

		Oscar		
		SS	SF	q-mix
Felix	RR	0	1	1-q
	RP	1	0	q
	p-mix	1-p	p	

If you are worried about eliminating weakly dominated strategies, here is a more rigorous argument. Felix's PP is strictly dominated by any mixture between RR and RP. With PP gone, Oscar's FF and FS become strictly dominated by SS or SF. This leaves a 3-by-2 matrix with RR, RP and PR for

Felix and SS and SF for Oscar. Now there is no more strict dominance. But PR is weakly dominated by RR, with a tie only in case Oscar is playing pure SF. So PR can be played with positive probability in a Nash equilibrium only if Oscar plays pure SF in that equilibrium. In that case, Felix's best response can be any mixture between RR and PR (with zero probability of playing RP). But against all such mixtures, Oscar's best choice is pure SS, not SF at all. Therefore there cannot be such a Nash equilibrium. This (essentially rationalizability) argument eliminates PR, and leaves the 2-by-2 matrix above.

The equilibrium of this is easy to calculate. Both players use mixed strategies. Felix plays each of RR and RP with probability 0.5, and Oscar plays each of SS and SF with probability 0.5. In other words, Felix always Raises when he draws High, and Raises with probability 0.5 when he draws Low. Given that Felix has Raised, Oscar always Sees when he draws the High card, and Sees with probability 0.5 when he draws the Low card.

(d) Felix's decision reveals some information about his card (High or Low), and he has an advantage when he draws High. If Felix always Raises when he draws High and always Passes on Low, then Oscar can draw the appropriate inferences: if Felix has raised he must have a High card. Oscar can then defend optimally, and then Felix's expected payoff would be only 0. Fully revealing his card draw thus limits Felix's winnings. Felix must sometimes (but not always) Raise when he draws Low in order to avoid exploitation. Using this mixed strategy allows him to (partly) conceal his private information; this is signal jamming, or bluffing. Then his expected payoff is positive (0.5 in equilibrium).

Some discussion of bluffing:

In a poker game, *bluffing* means betting as though one has a good combination of cards in one's hand when one really doesn't. Bluffing has an obvious disadvantage; if your opponent isn't scared out of the game, your increased bet may lead to an increased loss. Bluffing also has two possible advantages.

If bluffing causes the other players to drop out of the game, it allows you to win with a weak hand. Also, when you have a good hand and the other players know that you sometimes bluff, you may be able to increase your bet without leading the other players to drop. When you hold a good hand, therefore, your past bluffing may allow you to win more than you would have otherwise. The goal of bluffing is to mislead your opponent about the strength of your hand (your hidden information), or at least to make it difficult for the opponent to infer your strength.

SOLUTION USING BAYES' RULE OR FORMULA

Drawing inferences about Felix's cards by observing his actions is an application of Bayes' rule or formula in probability theory. An exposition is in *Games of Strategy*, Chapter 9, Appendix Section 1, pp. 301-303. In our context, this works as follows. Felix can condition his action on the observation of his own card, and in general can use mixed strategies. So write P_H for the probability that he chooses Raise when his card is High, and P_L for the probability that he chooses Raise when his card is Low. His getting a High card and a Low card are equally likely (probability $\frac{1}{2}$ each). Therefore we can show the following "probability table" (see Figure 9A-1 in the book); this is at the top of p.5.

As an example of drawing inferences from this, suppose Oscar sees that Felix has Raised. Then he knows that an event of probability $(\frac{1}{2} P_H + \frac{1}{2} P_L)$ has occurred. Conditional on this, or within the set of possibilities this comprises, the probability that Felix has a high card is

$$\frac{1}{2} P_H / (\frac{1}{2} P_H + \frac{1}{2} P_L) = P_H / (P_H + P_L) .$$

Other inferences can be drawn similarly.

		Felix's action		Sum of row
		Raise	Pass	
Felix's card	High	$\frac{1}{2} P_H$	$\frac{1}{2} (1-P_H)$	$\frac{1}{2}$
	Low	$\frac{1}{2} P_L$	$\frac{1}{2} (1-P_L)$	$\frac{1}{2}$
Sum of column		$\frac{1}{2} P_H + \frac{1}{2} P_L$	$\frac{1}{2} (1-P_H) + \frac{1}{2} (1-P_L)$	

Now we can examine what kind of strategies can constitute a Nash equilibrium of the extensive form game. The most general strategies are mixed:

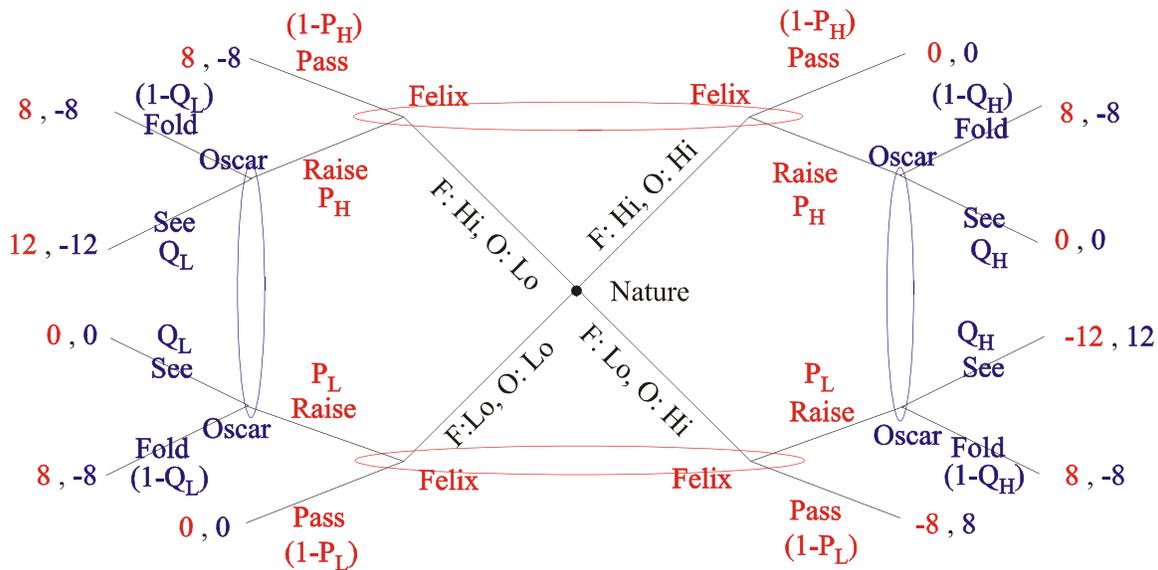
Felix: When own card is High, choose Raise with probability P_H , Pass with probability $(1-P_H)$

When own card is Low, choose Raise with probability P_L , Pass with probability $(1-P_L)$

Oscar: When own card is High, choose See with probability Q_H , Fold with probability $(1-Q_H)$

When own card is Low, choose See with probability Q_L , Fold with probability $(1-Q_L)$

What is needed for equilibrium? [1] Each player's strategies should be optimal choices at every information set (subgame perfectness). [2] Oscar should be drawing correct (Bayes' Rule) inferences from the observation that Felix has raised. Remember that Oscar only gets to choose if Felix has chosen Raise. To help check these things, here is the game tree with the mixture probabilities shown:



Let us consider the best responses, where each player makes a purposive choice of his own actions (or mixture), taking the choice probabilities of the other as given.

Suppose Felix has Raised, and consider Oscar's best response. As we calculated above, Oscar infers that the probability of Felix having a High card is $P_H / (P_H + P_L)$, and then that of Felix having a Low card is $P_L / (P_H + P_L)$. Oscar knows what card he himself holds. If it is High (the two nodes in Oscar's information set on the right hand side), he assigns these probabilities to the two nodes, and

calculates the expected payoffs:

$$\text{Fold: } -8 P_H / (P_H + P_L) - 8 P_L / (P_H + P_L) = -8$$

$$\text{See: } 0 P_H / (P_H + P_L) + 12 P_L / (P_H + P_L) = 12 P_L / (P_H + P_L) .$$

Obviously See is better; so we know that in equilibrium we must have $Q_H = 1$. If, on the other hand, Oscar is holding the Low card (the two nodes in his information set on the left hand side), the corresponding calculations yield

$$\text{Fold: } -8 P_H / (P_H + P_L) - 8 P_L / (P_H + P_L) = -8$$

$$\text{See: } -12 P_H / (P_H + P_L) + 0 P_L / (P_H + P_L) = -12 P_H / (P_H + P_L) .$$

Now matters are not so clear. Oscar should choose See if

$$-12 P_H / (P_H + P_L) > -8 , \text{ or } 12 P_H / (P_H + P_L) < 8 ,$$

$$\text{or } 12 P_H < 8 (P_H + P_L), \text{ or } 4 P_H < 8 P_L , \text{ or } P_H < 2 P_L .$$

Roughly speaking, if P_H is low in relation to P_L , then Felix's raising still leaves a sufficiently high probability that Felix's card is Low, making it worthwhile for Oscar to See even if his own card is Low, thereby taking a chance of getting 0 or -12 in preference to a certainty of -8 which would come from choosing Fold. Conversely, with a Low card Oscar should choose Fold if $P_H > 2 P_L$. If $P_H = 2 P_L$, then Oscar with a Low card is indifferent between his two choices, or any mixture of the two.

Next consider Felix's choice. He sees his own card, and has no actions of Oscar available for drawing any inferences, so he regards the probabilities of Oscar's card being High and Low as $\frac{1}{2}$ each. Suppose Felix's card is High (the top information set). He calculates expected payoffs as follows:

$$\text{Pass: } \frac{1}{2} 8 + \frac{1}{2} 0 = 4$$

$$\text{Raise: } \frac{1}{2} [12 Q_L + 8 (1-Q_L)] + \frac{1}{2} [0 Q_H + 8 (1-Q_H)] = 4 + 2 Q_L + 4 Q_H$$

Therefore Felix with a High card should Raise if $4 + 2 Q_L + 4 Q_H > 4$, or $2 Q_L + 4 Q_H > 0$, which is true.

So we know that in equilibrium $P_H = 1$. If Felix holds a Low card (the bottom information set), the expected payoffs are

$$\text{Pass: } \frac{1}{2} 0 - \frac{1}{2} 8 = -4$$

$$\text{Raise: } \frac{1}{2} [0 Q_L + 8 (1-Q_L)] + \frac{1}{2} [-12 Q_H + 8 (1-Q_H)] = 8 - 10 Q_H - 4 Q_L$$

Therefore Felix with a Low card should Raise if $8 - 10 Q_H - 4 Q_L > -4$, or $10 Q_H + 4 Q_L < 12$. Roughly speaking, smaller probabilities that Oscar chooses See (with either card) make it more attractive for Felix to Raise with a Low card, creating the possibility of getting the 8 when Oscar Folds.

Now combine all our calculations so far. We know that $P_H = 1$ (Felix chooses Raise for sure with a High card) and $Q_H = 1$ (Oscar chooses See for sure with a High card). Using these, we also have:

Felix with a Low card chooses

pure Raise ($P_L = 1$) if $10 + 4 Q_L < 12$, or $Q_L < \frac{1}{2}$,

pure Pass ($P_L = 0$) if $Q_L > \frac{1}{2}$,

and is indifferent (all P_L equally good) if $Q_L = \frac{1}{2}$.

Oscar with a Low card chooses

pure See ($Q_L = 1$) if $1 < 2 P_L$, or $P_L > \frac{1}{2}$,

pure Fold ($Q_L = 0$) if $P_L < \frac{1}{2}$,

and is indifferent (all Q_L equally good) if $P_L = \frac{1}{2}$.

The best response diagram for P_L and Q_L then shows that equilibrium must be mixed, with $P_L = \frac{1}{2}$ and $Q_L = \frac{1}{2}$.

