ECO 199 – GAMES OF STRATEGY
Spring Term 2004
PRECEPTS WEEK 8 (March 29-30)

We will discuss the following questions, to the extent that time permits:

Question 1

Consider the example of screening/signaling for skill from Games of Strategy, Chapter 9, Section 2, pp. 272-4. There were two types, A and C, with productivities and cost of taking each difficult course as follows

Type A: Productivity \$150K, cost per difficult course = \$6K

Type C: Productivity \$100K, cost per difficult course = \$9K

But now allow a third type B, whose productivity is \$125K and whose cost of taking each difficult course is \$7K. Find the incentive-compatibility conditions for sorting out the three types fully.

Question 2

Consider a more general version of Akerlof's Lemons market. Suppose the qualities of used cars in the population are uniformly distributed over the continuous interval (a, b), where $0 < a < b$. Suppose that a used car of any quality x is worth k times x in the hands of a buyer but only x in the hands of the seller, where $k > 1$. The quality of each used car is perfectly known to its current owner but cannot be observed at all by a potential buyer. And there are no effective signals. Can there be active trade in used cars? If so, at what price?

Question 3

In the movie *Dr. Strangelove, Or How I Learned to Stop Worrying and Love the Nuclear Bomb,* a crazy U.S. Air Force General, Jack D. Ripper, gives an uncalled-for and unauthorized order to the squadron of B-52 bombers under his command to go to attack their designated targets in the Soviet Union. American President Merkin Muffley phones the Soviet Premier Dimitri Kissoff to tell him what has happened, and offers information to help the Soviets shoot down the attacking USAF planes. Kissoff is suspicious, and asks "Is this a friendly call?" Muffley assures him: "Of course it is a friendly call. If it wasn't friendly, I would never have called." Discuss this from the perspective of strategic information communication.

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PRECEPTS WEEK 8 – SOLUTIONS

Question 1

There are six conditions, to rule out the possibility of each of the three types mimicking any of the other two. As before, the number of hard courses for the C-types is zero - if you are going to be taken for the worst type, there is no point incurring any signaling cost. Suppose the number needed to be thought a B-type is b, and that for being regarded an A-types is a. Then

For A not to mimic	B: $150 - 6a > 125 - 6b$,	or	$a - b < 25/6 = 4.2$
...	C: $150 - 6a > 100$,	or	$a < 50/6 = 8.3$
For B not to mimic	A: $125 - 7b > 150 - 7a$,	or	$a - b > 25/7 = 3.6$
...	C: $125 - 7b > 100$,	or	$b < 25/7 = 3.6$
For C not to mimic	A: $100 > 150 - 9a$,	or	$a > 50/9 = 5.6$
...	B: $100 > 125 - 9b$,	or	$b > 25/9 = 2.8$

If a and b are required to be integers, then the only values satisfying all the incentive compatibility constraints are $a = 7$ and $b = 3$. (If a and b can be general real numbers, then we can choose b (slightly greater than) 2.8 and a (slightly greater than) 6.4 .)

Observe the following:

(1) If a requirement is too small, a worse type will mimic a better type. So lower bounds on a requirement serve to rule out the mimicking of a better type. If a requirement is too large, a better type will mimic a worse type. So upper bounds on a requirement serve to rule out the mimicking of a worse type.

(2) If C does not mimic B ($b > 25/9 = 2.8$) and B does not mimic A ($a - b > 25/7 = 3.6$), then

$$a = b + (a-b) > 25/9 + 25/7 > 25/9 + 25/9 = 50/9 = 5.6,$$

so it follows that C will not mimic A and that condition is redundant. That is, only successive pairs of adjacent incentive-compatibility conditions need to be checked; those pertaining to types farther away from each other then follow automatically. This is because the cost of signaling for the successive types is increasing steadily; this is called the "Mirrlees-Spence single-crossing property", after two of the pioneers in the research on screening and signaling.

Likewise, in the other direction, $a - b < 25/6 = 4.2$ and $b < 25/7 = 3.6$ together ensure

$$a = b + (a-b) < 25/6 + 25/7 < 25/6 + 25/6 = 50/6 = 8.3.$$

But this is less important because the lowest feasible a and b are the most efficient so the upper limits do not usually come into play.

(3) In the book, there was no type B, and $a = 6$ sufficed to separate the type A from

mimicking by type C. Now, with an additional and closer type B who has a lower cost of mimicking A, and who in turn has to be separated from the old C, we need $a = 7$ to separate A from B.

Question 2

Suppose trade occurs at price p (which must be the same for all used cars because prospective buyers cannot distinguish between any two used cars). First suppose $p < b$; we will check when this assumption is valid and what happens if it is violated. Then only owners of cars with quality less than p will sell. The average quality of used cars on the market is therefore $(a+p)/2$. In the hands of the buyers, this is worth $k\,(a+p)/2$. In a competitive market, this is what buyers will have to pay, that is, $p = k\,(a+p)/2$, or

$$p = a\,k\,/\,(2-k).$$

This will be $< b$ as we started out by supposing, if $a\,k\,/\,(2-k) < b$, or $(a+b)\,k < 2\,b$, or $k < 2\,b/(a+b)$.

Note that $k > 1$ ensures $k/(2-k) > 1$, we have $p > a$ so long as $a > 0$. So a complete collapse of the market will occur only if $a = 0$, that is, the worst conceivable quality car is totally useless.

If $k \geq 2\,b/(a+b)$, our formula gives $p \geq b$. Then all owners will put their cars on the market. The quality of the average car will be $(a+b)/2$, and competition among buyers will lead to the price $p = k\,(a+b)/2$. Thus the previous formula for the price must be changed. We must now check that the new formula does give a price $\geq b$. And indeed, the inequality $k\,(a+b)/2 \geq b$ is the same as $k \geq 2\,b/(a+b)$, the starting point of the case under consideration.

The same inequality can be written in a more informative way as $b/a < k/(2-k)$, that is, the spread of qualities is not too large in relation to the premium in value which the buyer can achieve.

Question 3

This is intended to be a kind of signal. President Muffley's argument is in essence as follows: "Suppose there are two possible types of U.S. presidents - those who intend to destroy the Soviet Union and those who are willing to pursue a policy of coexistence with the Soviets. The former (bad) type would let Ripper's sneak attack proceed, and perhaps even reinforced it with more bombers, and not warn the Soviets at all. So the fact that I am warning you proves that I am of the latter (good) type."

In advance game-theory jargon, this kind of signal is sometimes called "forward induction." The idea is that one's current action is proof of one's intentions about choice of action in a future subgame.

The validity of this argument depends on the kinds of actions that are in principle available to the U.S. For example, there may be an even higher level of sneakiness, where the warning, and the offer of the help to shoot down this set of planes, lull the Soviets into a false sense of security and focus their attention on this particular danger, and then the U.S. launches a real attack in some other form and from some other direction. So Premier Kissoff was perhaps right to remain suspicious.