ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Final Examination

Important instructions:

[1] This is a closed-book examination. Put away your books, handouts, notes, computers, calculators, cellular phones, ... **now**.

[2] The exam has **four pages**. Make sure you have them all.

[3] The exam has three sections, labeled A, B and C. **Use a separate answer-book for each section.**

[4] **Print** your name clearly on the front cover of the exam and of each answer-book. No (or unclear) information – no grade.

[5] Write out and sign the honor pledge – "I pledge my honor that I have not violated the honor code during this examination" – on the front cover of your answer-book for Section A.

[6] Each question indicates its total point score, and the suggested time for your answer. Allocate your time optimally. Extra time can be "bought" at the rate of 4 points per minute or fraction thereof.

[7] Write clearly. Illegible answers will cost you points. Show the steps of your math. Use well-labeled diagrams and explain any symbols or notation you introduce.

Section A

Answer **both** of Questions 1–2

Question 1: (15 points, 25 minutes)

Define any five of the following terms, using a diagram or equation where appropriate.

(a) Principle of complementary slackness
(b) Utility function
(c) Pareto efficiency (also called Pareto optimality)
(d) Deadweight loss from monopoly
(e) Nash equilibrium
(f) Expected utility
(g) The winner's curse

Question 2: (15 points, 30 minutes)

Choose any three of the following concepts. For each, explain in a brief paragraph (about 50-60 words), with an accompanying equation or diagram where appropriate, its significance in microeconomic theory. You don't have to give any proofs.

(a) Lagrange multiplier
(b) Quasi-linear preferences
(c) Sunk costs
(d) Strategic commitment
(e) Individual or idiosyncratic risk

Section B

Answer any two of Questions 3–5

Question 3: (20 points, 35 minutes)

In this question, there are n goods whose quantities are denoted by X_1, X_2, ... X_n and prices by P_1, P_2 ... P_n. In parts (b)-(d) below, i and j can go from 1 to n. Money income is denoted by I. A utility-maximzing consumer is choosing the quantities taking the prices as given. The Hicksian demands are denoted by the superscript H, the Marshallian demands by the superscript M. You are told that all quantities are positive.

(a) State Shepherd's (also called Hotelling's) Lemma.
(b) State the Slutsky equation for the effects of a change in P_j on X_i .
(c) Prove that, for any two goods i and j :

$$\frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_j^H}{\partial P_i}$$

(d) Prove that for $i \neq j$, the equation

$$\frac{\partial X_i^M}{\partial P_j} = \frac{\partial X_j^M}{\partial P_i}$$

will be true if and only if

$$\frac{I}{X_i^M}\,\frac{\partial X_i^M}{\partial I} = \frac{I}{X_j^M}\,\frac{\partial X_j^M}{\partial I}$$

Question 4: (20 points, 35 minutes)

Consider the production function of a firm:

$$Q = [\, K^{1/3} + 2\, L^{1/3}\,]^4$$

where Q is output, K is capital input, and L is labor input. Answer the following questions. (Reminder: show the steps of your work.)

(a) Does this have increasing, constant, or decreasing returns to scale?

(b) Does it have a diminishing marginal rate of input substitution?

(c) The firm can hire capital and labor at given prices r and w respectively, and minimizes cost of producing each quantity Q. Express the ratio of its cost-minimizing input choices, L/K, as a function of the input price ratio r/w. Find the elasticity of substitution between capital and labor.

(d) Show that the firm's dual cost function is

$$C^*(r, w, Q) = Q^{3/4}\, [\, r^{-1/2} + 2^{3/2}\, w^{-1/2}\,]^{-2}\, .$$

(Hint: Use this given answer as a guide for collecting and simplifying expressions in the steps of your algebra.)

Question 5: (20 points, 35 minutes)

The time is just before the 2003 World Series. Betting slips (Arrow-Debreu securities) are being traded in a perfectly competitive market. Let p denote the price in today's market of a slip that pays \$1 if the New York Yankees win; with negligible discounting, $(1 - p)$ is the price in today's market of a slip that pays \$1 if the Florida Marlins win.

Nina Yang is a Yankees fan, and Flora Miller is a Marlins fan. Each has a logarithmic von Neumann-Morgenstern utility function. But Nina believes firmly that the probability that the Yankees will win the series is 90%, while Flora believes equally firmly that the probabilitiy that the Yankees will win the series is only 30%. Each has an initial wealth of \$10000, and this is not directly affected by the outcome of the series, so each has an initial endowment of 10000 Arrow-Debreu securities for each scenario. Each is a price-taker in the markets for the Arrow-Debreu securities, and they are the only traders in this market. (Think of each as a representative of several similar traders in these markets.)

(a) Write down their expected utility functions.

(b) Write down their budget constraints for trading in the Arrow-Debreu securities markets.

(c) Write down their demand functions for the Arrow-Debreu securities.

(d) Find the equilibrium value of p.

(e) Find the equilibrium betting choices of the two.

Section C

Answer **any one** of Questions 6–7

Note – Remember that the choice between the essay (Question 6) and the mathematical problem (Question 7) involves a risk-return tradeoff. A complete correct answer to the mathematical problem gets the full 30 points, but errors can drag you down far. It is very difficult to score anywhere close to 30 on the essay, because it will be graded for organization and clarity of the writing as well as it logical correctness and completeness. But so long as you include most of the relevant points and don't make serious mistakes, it is hard to drop below 15. So make your choice in the light of this information, and your own knowledge of your relative skills and attitudes to risk.

Question 6: (30 points, 55 minutes)

Explain, using a couple of illustrative examples each, the concepts of *moral hazard* and *adverse selection*. State briefly how moral hazard affects the functioning of competitive insurance markets. Explain in greater detail, using appropriate diagrams, how adverse selection affects the functioning of competitive insurance markets. For the present purpose, you should assume that insurance companies have negligible administrative or operating costs and can be risk-neutral by pooling numerous individual risks.

Question 7: (30 points, 55 minutes)

(Note – In this question you should do your arithmetic in fractions using paper and pencil methods – no calculators.)

Consider a duopoly where the quantities are denoted by Q_1, Q_2 and the prices by P_1, P_2. The (direct) demand functions are

$$Q_1 = 20 - 2\,P_1 + P_2, \qquad Q_2 = 20 + P_1 - 2\,P_2\,,$$

so the inverse demand functions are

$$P_1 = 20 - \frac{2}{3}\,Q_1 - \frac{1}{3}\,Q_2, \qquad P_2 = 20 - \frac{1}{3}\,Q_1 - \frac{2}{3}\,Q_2\,.$$

(You are told this; you don't have to solve one to prove the other.) The marginal cost of production of each good is constant and equal to 5. There are no other costs. Each firm wants to maximize its profit.

(a) Are the goods substitutes or complements? Why?

(b) Find the prices, quantities, and profits in the Bertrand-Nash equilibrium.

(c) Find the quantities, prices, and profits in the Cournot-Nash equilibrium.

(d) Next suppose the firms collude and choose prices to maximize their joint profit. Find the resulting prices, quantities, and profits.

(e) Finally, suppose the firms collude and choose quantities to maximize their joint profit. Find the resulting quantities, prices, and profits.

(f) Rank the prices, quantities and profits you found in the alternative situations (b)-(e) above. *Briefly* state the economic intuitions for the differences.
