ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Midterm Examination Answer Key

This was graded by Arnaud Costinot, and the distribution was as follows:

Range	Number
100	1
90–99	5
80–89	14
70–79	12
60–69	13
50–59	17
40–49	10
30–39	7
< 30	7

The mean and the median were both 61, the standard deviation was 20.

On the whole, the performance was slightly worse than in some previous years. A particularly disappointing aspect was that too many of you did not remember the definitions of basic concepts. But there is plenty of time to improve. As an extra incentive, I will change the formula for computation of the course grade. If your score on the final exam is higher than your score on the midterm exam, your final exam will count for 60% (instead of 50), and your midterm for only 10% (instead of 20). If your midterm score is higher than your final score, the usual weighting (20% for the midterm and 50% for the final) will apply.

Question 1 – 20 points

Definitions – 5 points each for the four you answered. If you answered more, the best four were chosen. Points get deducted if your definition is incomplete or unclear; the deduction depends on the seriousness of the deficiency or lack of clarity. I have given below the exact wording in the end-of-chapter "Key Concepts" in the textbook.

COMMON ERRORS: (1) Elasticity of substitution: Many people just gave the general definition of an elasticity (of Y with respect to X,s ay), and not the specific one needed. (2) Confusion between decreasing returns to scale and decreasing returns to one factor. (3) Saying that diminishing MRS means a downward-sloping indifference curve (instead of the correct convex indifference curve). (4) Saying that the dual cost function was $C = wL + rK$, when what is needed is the minimum of this with respect to L and K, which then becomes a function of (w, r, Q).

(a) (Textbook pp. 110, 133, Lecture handout Sep. 23, p. 2) Preferences are transitive if, for all triples of consumption bundles A, B, C, "A preferred to B" and "B preferred to C" together imply "A preferred to C".

(b) (Textbook pp. 115, 133, Lecture handout Sep. 23, p. 2) Preferences are said to exhibit diminishing marginal rate of substitution if indifference curves are smooth and convex to the origin (book) OR if, as x increases and y decreases along an indifference curve $U(x, y) =$ constant, the numerical value $-dy/dx$ of the marginal rate of substitution decreases (lecture handout).

(c) (Textbook pp. 184, 211, Lecture handout Sep. 25, p. 1) The indirect utility function describes the maximum utility for every combination of income and prices. It is the solution to the utility maximization problem, expressed as a function of income and prices. Alternatively, you could just give the mathematical formula

$$U^*(P_x, P_y, M) = \max \{ U(X, Y) \mid P_x X + P_y Y \leq M \}$$

and a brief statement of what the symbols stand for.

(d) (Textbook pp. 196-7, 211-2, Precept Materials Week 3 Question 3) The elasticity of substitution (the book says along an indifference curve, which is the more correct formulation, but I obscured the distinction so it is OK if you did) measures the percentage change in the ratio of y purchases to x purchases in response for each unit of percentage change in the price ratio p_x/p_y. You can, but don't have to, give the mathematical expression

$$\frac{p_x/p_y}{x/y} \left. \frac{d(y/x)}{d(p_x/p_y)} \right|_{\text{u = constant}} = \left. \frac{d \ln(y/x)}{d \ln(p_x/p_y)} \right|_{\text{u = constant}}$$

(e) (Textbook pp. 250, 257, Lecture handout Oct. 9 p. 3) A production function has decreasing returns to scale if doubling all inputs less than doubles output (homogeneity of degree less than 1). This is the definition in the book and it is somewhat less precise than the one I have: For any K, L and any scaling factor $s > 1$, the function satisfies $F(sK, sL) < s\,F(K, L)$ (so the function does not have to be homogeneous at all). But either is acceptable in your answer.

(f) (Textbook pp. 264, 298, Lecture handout Oct. 14, pp. 1-2) A firm's (dual) cost function describes the lowest possible economic cost to produce each output level. It is a function of the output quantity and the input prices. Alternatively, the mathematical formula and a statement of what the symbols stand for:

$$C^*(r, w, Q) = \min \{ r\,K + w\,L \mid F(K, L) \geq Q \}$$

Question 2 – 30 points

Each part carries 15 points. If you answered all three, the best two are chosen.

COMMON ERRORS: (1) Not clarifying notation, and not spelling out the steps of the argument. For example, in deriving the Slutsky equation, $\partial M^*/\partial P_j$ is the compensated demand quantity of good j, which is then equal to the uncompensated quantity because the money and the utility in the two problems are "just right" for each other. These two steps must be shown, and not collapsed into one. We were quite tough in deducting points for these omissions. (2) When using the chain rule of differentiation, many of you did not correctly show which derivatives were total and which ones were partial, but no points were taken off for this if the rest of the argument was fully spelled out. (3) The discussion of empirical work was generally very poor. Some of you thought that heterogeneity of preferences in the population is a violation of utility theory. No; utility theory lets each person have his/her own preferences, subject only to the requirement of internal logical consistency (completeness and transitivity). No one mentioned the animal experiments discussed in the book.

(a) Textbook pp. 74-5 or Lecture handout Sep. 16 p. 5. Here is a repeat of the latter, showing the steps of the derivation you have to give: Objective function $F(x,y)$, constraint $G(x,y) = c$.

Lagrangian: $L(x,y,\lambda) = F(x,y) + \lambda\,[c - G(x,y)]$

FONCs

$$\begin{aligned} L_x(x,y,\lambda) &= F_x(x,y) - \lambda\; G_x(x,y) = 0 \\ L_y(x,y,\lambda) &= F_y(x,y) - \lambda\; G_y(x,y) = 0 \end{aligned}$$

Solution (x^*, y^*). (You need to state this so the grader can understand your notation. Three points off if you omit this or a good equivalent statement.)

Let $v = F(x^*, y^*)$, all are functions of c.

$$\begin{aligned} dv/dc &= dF(x^*,y^*)/dc \\ &= F_x(x^*,y^*)\,dx^*/dc + F_y(x^*,y^*)\,dy^*/dc \quad \text{by chain rule} \\ &= \lambda\,G_x(x^*,y^*)\,dx^*/dc + \lambda\,G_y(x^*,y^*)\,dy^*/dc \quad \text{by FONCs} \\ &= \lambda\;dG(x^*,y^*)/dc \quad \text{by chain rule} \\ &= \lambda\;dc/dc = \lambda \end{aligned}$$

(b) Textbook pp. 192-3, Lecture handout Sep. 25 p. 6. The book does this only for the quantity of the same good as the one whose price changes; I did it for the more general case of the effect of one price change on the quantity of that or any other good. OK if you did it as in the book, or for the case of just two goods. Here is a repeat of the lecture handout showing the steps you have to give.

Notation: $(P_1, P_2, \ldots)$ prices, $(X_1, X_2, \ldots)$ quantities. u denotes the utility level and $M^*(P_1, P_2, \ldots, u)$ denotes the expenditure function. $D^H(P_1, P_2, \ldots, u)$ denotes Hicksian demands and $D^M(P_1, P_2, \ldots, M)$ denotes Marshallian demands. Three points off if you do not explain your notation clearly.

For good i where i may be either x or y,

$$D_i^H(P_x, P_y, u) = D_i^M(P_x, P_y, M^*(P_x, P_y, u)) \qquad (1)$$

Now let P_j change. Differentiate the above equation

$$\begin{aligned} \frac{\partial D_i^H}{\partial P_j} &= \frac{\partial D_i^M}{\partial P_j} + \frac{\partial D_i^M}{\partial M}\,\frac{\partial M^*}{\partial P_j} \quad \text{by chain rule} \\ &= \frac{\partial D_i^M}{\partial P_j} + \frac{\partial D_i^M}{\partial M}\,D_j^H \quad \text{by Shepherd's Lemma} \\ &= \frac{\partial D_i^M}{\partial P_j} + \frac{\partial D_i^M}{\partial M}\,D_j^M \quad \text{by (1) above} \end{aligned}$$

(c) You should mention the following: (i) Textbook p.131 – Experiments on animals present them with various "budget constraints" and observe their choices. They are broadly found to conform to the requirements of revealed preference. (ii) Lecture handout Oct. 7, p. 4. Experiments, usually on college student subjects, find anomalies. Choice depends on the design of the procedure and on the status quo, not just the final consumption quantities. Endowment effects are most prominent. There are also dynamic inconsistencies of choice, where the tradeoff between consumption at dates 2 and 3 looks very different on date 1 than it does when date 2 actually arrives. (iii) Lecture handout Sep. 30. Econometric work on actual purchase data using large household-level data sets and good statstical techniques finds reasonable estimates of income and price elasticities. But some violations of homogeneity and Slutsky symmetry. Each of these items gets 5 points; within each, the points depend on how complete is your answer.

Question 3 – 50 points

COMMON ERRORS: (1) Wrong budget constraints, for example forgetting about the 80 for the first 8 hours when overtime is worked. (2) Not explaining why and how $H = 10$ gives a local and then the global max in part (b). More generally, insufficient explanation of steps. (3) Simple algebra errors. (3) Not checking SOSCs. Here we were generous: instead of deducting any points from the numerous people who did not check them, we gave +1 to the few people who did check.

(a) (5 points) If the wage rate is \$10 per hour, $C = 10\,H = 10$. This is the budget constraint. The utility function in terms of C and H is

$$U = \ln(C) + 3\,\ln(24 - H)$$

(15 points) Substitute from the budget constraint to write utility as a function of H alone:

$$U = \ln(10\,H) + 3\ \ln(24 - H) = \ln(10) + \ln(H) + 3\ \ln(24 - H)\,.$$

This has FONC

$$\frac{dU}{dH} \equiv \frac{1}{H} - \frac{3}{24 - H} = 0\,.$$

This yields $24 - H = 3\,H$, so $H = 6$. The SOSC is easy to check:

$$\frac{d^2U}{dH^2} = -\frac{1}{H^2} - \frac{3}{(24 - H)^2} = -\frac{1}{36} - \frac{3}{18^2} < 0.$$

The resulting utility is

$$U = \ln(10 \times 6) + 3\ \ln(18)\,.$$

Alternatively, you could use Lagrange.

$$L = \ln(C) + 3\ \ln(24 - H) + \lambda\,[\,10\,H - C\,]\,.$$

Therefore the FONCs are

$$1/C - \lambda = 0, \quad -3/(24 - H) + 10\,\lambda = 0\,.$$

Then

$$0 = C - 10\,H = 1/\lambda + 3/\lambda - 240.$$

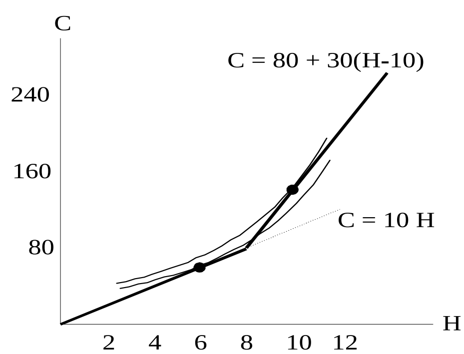
So

$$1/\lambda = 60, \qquad \text{then} \qquad C = 60, \quad \text{and} \quad 24 - H = \tfrac{3}{10}\,60 = 18, \quad \text{so} \quad H = 6.$$

Therefore the consumer works $H = 6$ hours, and the resulting utility is

$$U = \ln(60) + 3\ \ln(18)\,.$$

(b) You should recognize the economics of this situation. The consumer gets paid an overtime rate if he chooses to work more than 8 hours. The previous choice of 6 hours remains a local optimum (within the interval from 0 to 8 hours). The question is whether there is another local optimum where he does work overtime. As usual, a rough sketch helps you understand what is going on (although you were not required to draw one here). The figure below is more accurate than you need. Note that the horizontal axis has hours of work H, which is a "bad". Its opposite, $N = 24 - H$, is the "good". That is why indifference curves have the form shown – they are just the usual shape with the horizontal axis flipped around.



(5 points) Assume for the moment that the consumer does work overtime. Then the budget constraint is

$$C = 80 + 30\,(H - 8) = 30\,H - 160\,.$$

(20 points) Substituting this into the utility function (I will omit the equivalent Lagrange formulation)

$$U = \ln(30\,H - 160) + 3\ \ln(24 - H)\,.$$

The FONC is

$$\frac{dU}{dH} \equiv \frac{30}{30\,H - 160} - \frac{3}{24 - H} = 0\,.$$

This yields $720 - 30H = 90\,H - 480$, or $120\,H = 1200$, or $H = 10$. The SOSC for a local max is again easy to check:

$$\frac{d^2U}{dH^2} = - \frac{900}{(30\,H - 160)^2} - \frac{3}{(24 - H)^2} = - \frac{900}{140^2} - \frac{3}{14^2} < 0.$$

The solution $H = 10$ is actually > 8, so it gives a local max with overtime. It remains to check whether it is the global maximum. The utility with overtime work is

$$U^o = \ln(140) + 3\ \ln(14) = \ln(10) + 4\ \ln(14)\,.$$

(5 points) It remains to check whether this is greater than the utility at the other local optimum, namely that at the solution in part (a) without overtime work. Call that

$$U^n = \ln(60) + 3\ \ln(18)\,.$$

We have

$$U^o > U^n \text{ if and only if } 10 \times 14^4 > 60 \times 18^3 \text{ or } 14^4 > 6 \times 18^3\,.$$

Using the information given, this inequality becomes

$$38416 > 6 \times 5832 = 34992\,,$$

which is true. (Even if you can't do the arithmetic on the right hand side, you should see that $38416 > 36000$ and $6 \times 5832 < 6 \times 6000$.)