ECO 305 – Fall 2003

SUMMARY OF (SHORT RUN) COST CONCEPTS

Total Cost: $TC = C(Q)$

Fixed and Sunk (Unavoidable or Zeroeth-Copy) Cost: $FSC = C(0)$
Fixed but Non-Sunk (Avoidable or First-Copy) Cost: $FNC = \lim_{Q \downarrow 0}\ C(Q) - C(0)$
Fixed (Sunk plus Non-Sunk) Cost: $FC = FSC + FNC = \lim_{Q \downarrow 0}\ C(Q)$

Total Variable Cost: $TVC = TC - FC = C(Q) - \lim_{Q \downarrow 0}\ C(Q)$
Total Avoidable Cost: $TAC = TC - FSC = FNC + TVC = C(Q) - C(0)$

Average Total Cost: $ATC = TC/Q$
Average Avoidable Cost: $AAC = TAC/Q$
Average Variable Cost: $AVC = TVC/Q$
Marginal Cost: $MC = C'(Q)$

Firm's (short run) supply curve $S = MC$ when $p > AAC_{min}$, 0 if $p < AAC_{min}$

All are illustrated on the next page. The upper panel shows the total cost curve, and the lower panel shows various average and marginal cost curves. An average cost appears in the upper panel as the slope of the line joining the origin to a point on the total cost curve, and the marginal cost appears there as the slope of the total cost curve. Note how the lowest points of the various average cost curves line up with corresponding points on the total cost curve where the slopes of the lines from the origin are minimum. The lowest point of the marginal cost curve lines up with the point of inflection on the total cost curve.

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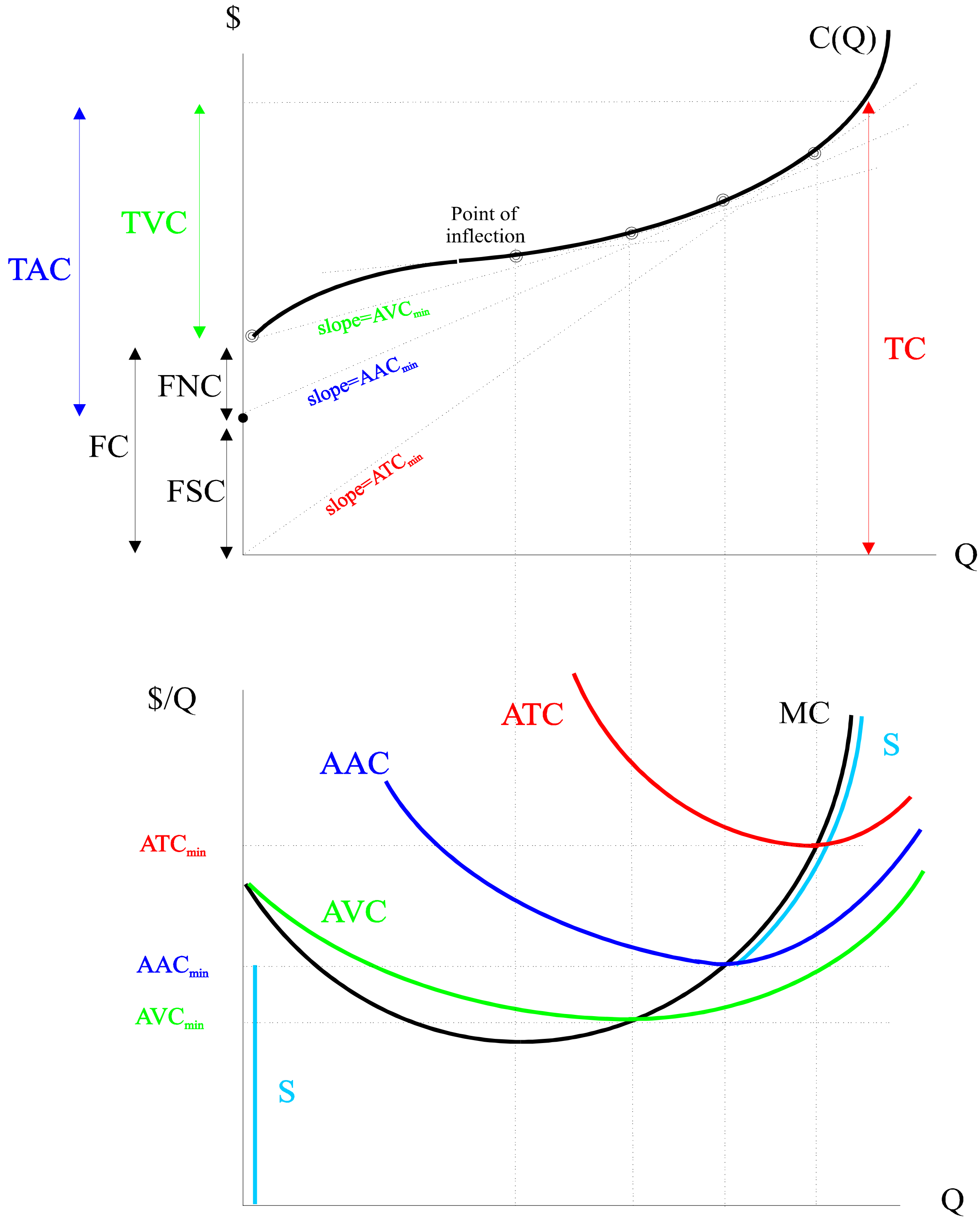

$
C(Q)
TVC
TAC
Point of
inflection
slope=AVCmin
slope=AACmin
slope=ATCmin
FNC
FC
FSC
TC
Q
$/Q
ATC
MC
S
AAC
ATCmin
AVC
AACmin
AVCmin
S
Q

EMPIRICAL EXAMPLES OF COST FUNCTIONS FUNCTIONS

There is an extensive literature on the empirical estimation of production, profit and cost functions. Discussions and surveys include Dale W. Jorgenson, *Productivity, Vol. 2,* MIT Press, 1995, and Ernst R. Berndt, *The Practice of Econometrics,* Addison-Wesley, 1991, Chapters 3, 9. In this course whose focus is theory, we can only offer a couple of examples.

Berndt (pp. 469-476) estimates a cost function for U.S. manufacturing as a whole. This involves additional issues of aggregation over numerous firms, which we must skip over. Output Y is produced using capital K, labor L, energy E, and other intermediate inputs M. The cost C is assumed to have the form

$$\begin{aligned} \ln C &= \ln(\alpha_0) + \sum_i \alpha_i \ln(P_i) + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln(P_i) \ln(P_j) \\ &\quad + \alpha_Y \ln Y + \tfrac{1}{2} \gamma_{YY} (\ln Y)^2 + \sum_i \gamma_{iY} \ln(P_i) \ln Y \end{aligned}$$

where the various subscripted Greek letters are parameters, the P_i are prices of the inputs, and the subscripts i and j run over the inputs K, L, E, and M. The following restrictions are imposed on the parameters (Why?):

$$\sum_i \alpha_i = 1, \qquad \gamma_{ij} = \gamma_{ji}, \qquad \sum_i \gamma_{ij} = 0 \text{ for } j = K,\, L,\, E,\, M,\, Y.$$

Actually the estimation is done not on the cost function itself, but on the factor cost share functions that can be obtained from it using Hotelling's (or Shepherd's) Lemma; for example the share of wages in cost is

$$\frac{P_L\, L}{C} = \frac{P_L}{C} \frac{\partial C}{\partial P_L} = \frac{d \ln C}{d \ln P_L}\,.$$

The estimates of the coefficients themselves are less interesting than their economic implications. Specifically, from the input demand functions we can calculate the elasticities of substitution σ_{ij} between any pair of inputs i and j. Berndt reports

$$\sigma_{KL} = 0.97,\ \sigma_{KE} = -3.60,\ \sigma_{KM} = 0.35,\ \sigma_{LM} = 0.61,\ \sigma_{EM} = 0.83,\ \sigma_{LE} = 0.68$$

Thus all distinct pairs of inputs are substitutes (an increae in the price of one increases the demand for the other) except capital and energy which are quite strongly complements. If the cost function were Cobb-Douglas, all these cross-substitution elasticities would equal 1; most pairs show less substitution, except capital and labor which is almost at its Cobb-Douglas level. The own price elasticities of demands for the inputs are

$$\epsilon_K = -0.34,\ \epsilon_L = -0.45,\ \epsilon_E = -0.53,\ \epsilon_M = -0.24\,.$$

These again show substantially less substitution than Cobb-Douglas (in which case all the numbers would equal 1).

Unfortunately γ_{YY} does not appear in the factor share equations (Why?), so Berndt does not estimate it and does not report whether his cost function shows increasing or decreasing returns to scale. But in Chapter 3 he surveys research on scale economies in U.S. electricity generation. The estimates show $\gamma_Y < 1$ and $\gamma_{YY} > 0$; thus the returns to scale are initially increasing for a range of small outputs but eventually decreasing for large scale (Why?). He also finds that the bulk of output was being produced "by firms operating in the essentially flat area of the average cost curve."

Stephen Moeller, in his 1999 Princeton senior thesis, *Cost Consequences of the Growth of U.S. Credit Unions: Information and Scale Effects,* estimates a cost function for credit unions. His specification is of the Cobb-Douglas form:

$$\ln C = a + b_1 \ \ln Q + b_2 \ (\ln Q)^2 + \sum_i c_i \ \ln W_i + \sum_j d_j \ \ln F_j + \mu \,,$$

where Q is a measure of the size (output) of the credit union, W_i are factor prices, F_j are other structural variables being controlled for, and μ is a stochastic error term. (Much thought and care is needed in choosing the appropriate measures of all these variables and then constructing the data sets; interested readers should consult the thesis for the details.) Here is a typical finding. Measuring the output by the total number of loans outstanding at the year's end, Moeller finds the following coefficient estimates for 1998:

$$\begin{aligned} b_1 &= 0.6537 && \text{with standard error } 0.0231, \\ b_2 &= 0.0204 && \text{with standard error } 0.0015. \end{aligned}$$

Thus both coefficients are very tightly estimated. Since $b_1 < 1$, there are economies of scale, but since $b_2 > 0$, these eventually run out. The intuition is that a larger credit union gains the benefits of better diversification and can spread its administrative overheads over a larger volume of business. But as it expands, it acquires a less coherent and more hetergeneous membership about which it has less information and over which it has less moral suasion; therefore it has a greater cost of default.

Averge cost is minimized when

$$\ln Q = (1 - b_1)/(2 \ b_2) = 8.48, \qquad \text{or} \qquad Q = 4764 \,.$$

He finds that 85% of U.S. credit unions were actually to the left of this point. The median union had $Q = 705$. However, the average cost curve was pretty flat near its bottom – comparing two unions that differ in size but are otherwise similar, the average cost of the median union was only 7.8 percent higher than that of the optimum-sized union.