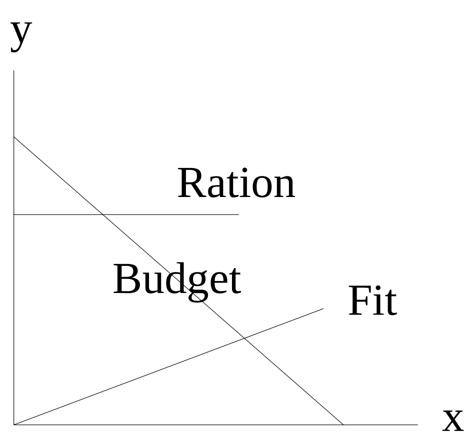
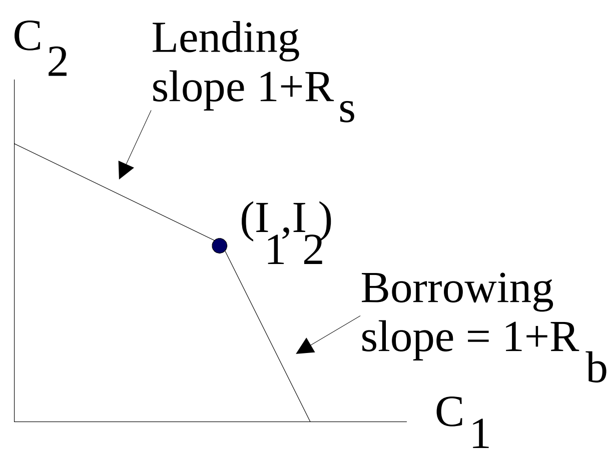
Many Variables and Constraints

Equation vs. inequality constraints

Budget line (equation): $P_x\,x + P_y\,y = I$
Inequality budget constraint: $P_x\,x + P_y\,y \leq I$
At optimum (x^*, y^*), constraint is called
"binding" or "tight" if $=$, "slack" if $<$
Number of equation constraints must be less than
number of choice variables
Any number of inequality constraints OK but
no. of binding constrs. $\leq$ no. of choice variables

Two example: (1) $x =$ food, $y =$ clothing
Budget: $P_x\,x + P_y\,y \leq I$, Ration: $y \leq R$, Fit: $y \geq k\,x$





(2) Rate of interest paid to borrow R_b
$>$ Rate of interest received on saving/lending R_s

Kuhn-Tucker Theory

Vector $\mathbf{x} = (x_1, x_2, \ldots x_n)$
Maximize $F(\mathbf{x})$ subject to $G_i(\mathbf{x}) \leq c_i$ ($i = 1$, 2, $\ldots$ m)
Lagrangian (with $\lambda_i \geq 0$)

$$L(\mathbf{x}, \lambda) = F(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i \left[c_i - G_i(\mathbf{x}) \right]$$

FONCs for $\mathbf{x}^*$ to be interior smooth local maximum:

$$\partial L / \partial x_i = 0 \text{ for } i = 1, 2, \ldots, n$$

Need another m equations to find the λ_i also.
Principle of Complementary Slackness
    If $G_i(\mathbf{x}^*) < c_i$, then $\lambda_i = 0$
    If $\lambda_i > 0$, then $G_i(\mathbf{x}^*) = c_i$

This agrees with previous interpretation
    of Lagrange multiplier as marginal increase
    in objective when constraint is relaxed:
A slack constraint has zero shadow price; a constraint
    with positive shadow price cannot be slack
Can have $G_i(\mathbf{x}^*) = c_i$ **and** $\lambda_i = 0$ in exceptional
    situations where constraint about to become slack

Which of these is true at the optimum remains to be found out, at worst by investigating all 2^m possible combinations one at a time.

Can also do non-negativity constraints using this theory.
All this sounds difficult, best learned by doing examples: many to come in class, precepts, problem sets, exams.

Topics from textbook chs. 2–3 omitted here:

Comparative Statics and Envelope Theorem
pp. 46–50, 75
Second-order conditions:
Appendixes pp. 54–55, 90–91
Will do specific versions of these in the context of economic applications; general theory not needed.

3-D pictures of constrained maximization and duality:
pp. 60, 63, 71–72, 77.
I am doing equivalent approach in 2-D using contours.

CONSUMER CHOICE THEORY – BASIC ISSUES

Choice-making unit – individual, household, . . . ?

Dimensions of choice – quantities of goods and services
 Labor supply (leisure demand) – income "endogenous"
 Borrowing or saving, portfolio choice
 Risk choices – purchase of insurance, gambling

Quantities taken to be continuous variables
 unless explicitly stated otherwise (rarely)

Constraints – budget line or nonlinear schedule
 because of quantity discounts or premia
 Other constraints like rationing

Time-span – If too short, whims and errors may dominate
 If too long, available goods, tastes may change

Economics – methodological individualism, rational choice
 Rationality – (1) internally consistent preferences
 (2) maximization of these subject to constraints
Preferences need not be selfish, purely money-oriented,
 short-run, conformist . . .
Maximization can be "as if"
Even then, should not take theory literally
 Look for explanation of average over people, time
 Judge success of theory by empirical evidence

Start simple and gradually build more complex models