COST-MINIMIZATION

$$C(w,r,Q) = \min\{\, w\,L + r\,K \mid F(K,L) \geq Q \,\}$$

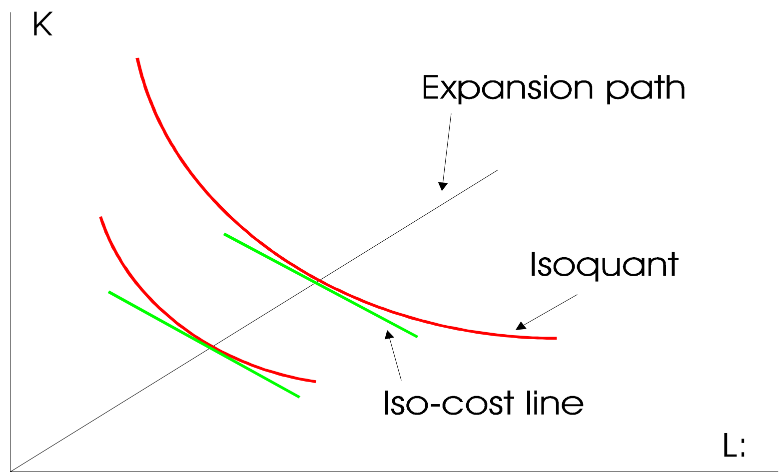
FONCs

$$w = \lambda\, \partial Q/\partial L, \quad r = \lambda\, \partial Q/\partial K$$

Interpretation: $\lambda =$ marginal cost

$$MRTS_{KL} = - \left.\frac{dK}{dL}\right|_{Q \text{ const.}} = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{w}{r}$$

Expansion path: Increase Q holding (w,r) fixed:



Production function *homothetic* if every
expansion path is a ray through origin

If F is homogeneous (of any degree a):
$F(sK, sL) = s^a\ F(K,L)$, then it is homothetic
Converse not true

PROPERTIES OF COST FUNCTION

Vary Q holding (w, r) fixed – (ECO 102 material)

Fixed cost: $C(0) = 0$, but as $Q \downarrow 0$, $\lim C(Q) > 0$

Sunk cost: $C(0) > 0$

$AC(Q) = C(Q)/Q$, $MC(Q) = C'(Q)$

Returns to scale $\uparrow$ at margin: $AC \downarrow$, $MC < AC$

returns to scale $\downarrow$ at margin: $AC \uparrow$; $MC > AC$

If rets to scale first $\uparrow$, then $\downarrow$, U-shaped cost curves

Vary (w, r) holding Q constant – (new material)

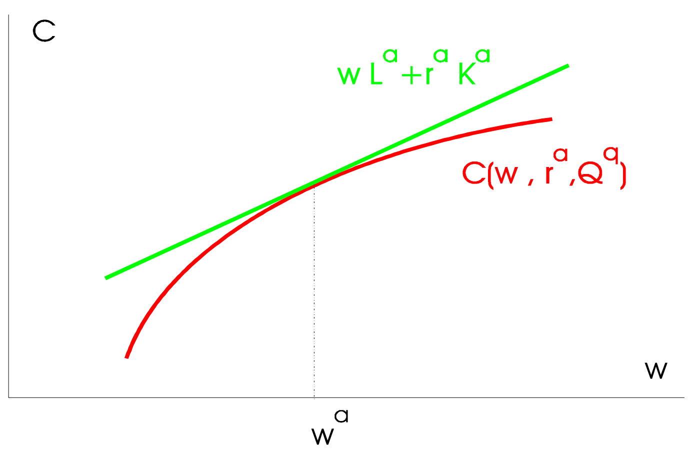
Properties similar to consumer's expenditure function

(1) Homogeneous degree 1. (2) Concave, and

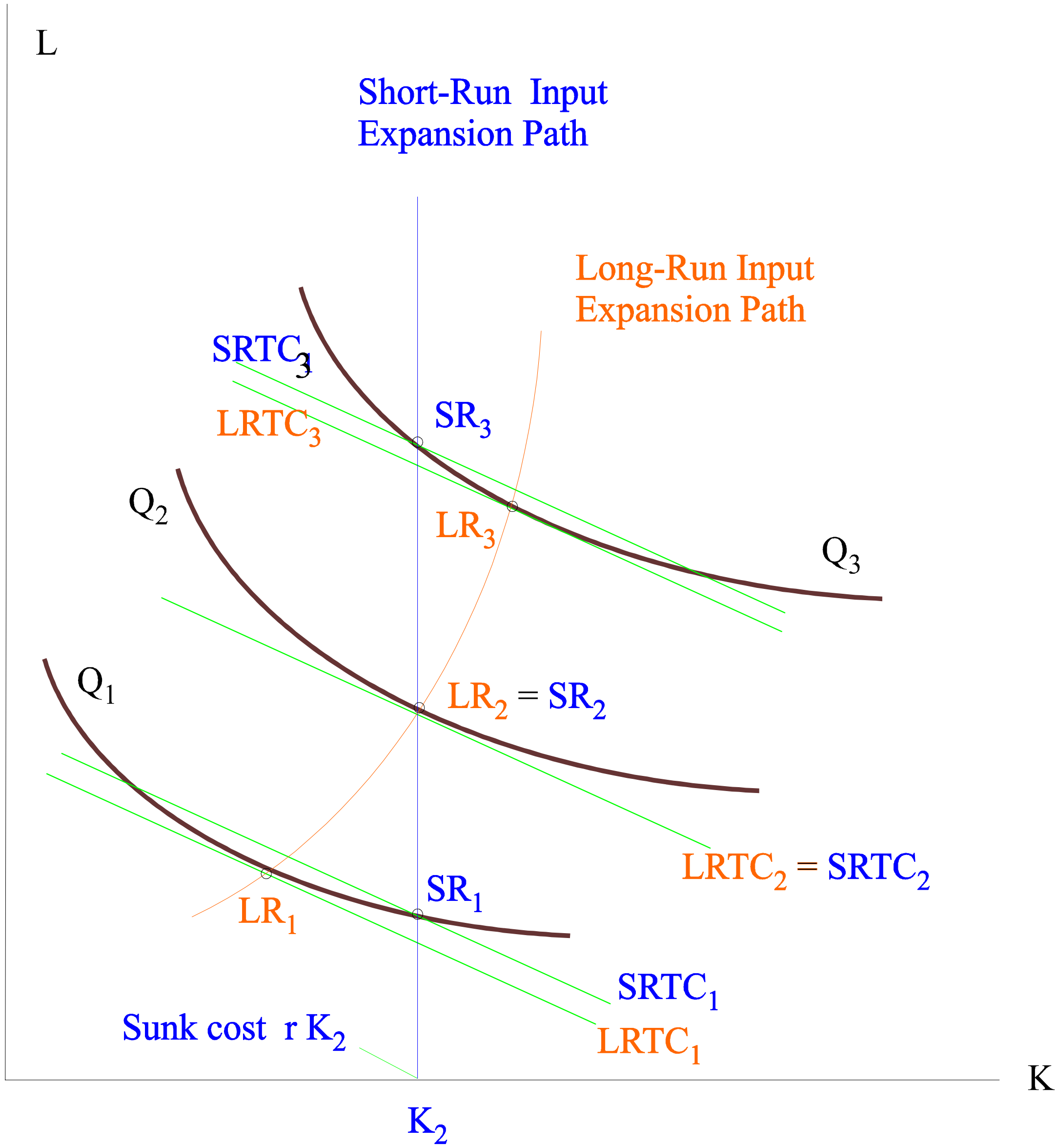
(3) Hotelling's (Shepherd's) Lemma,

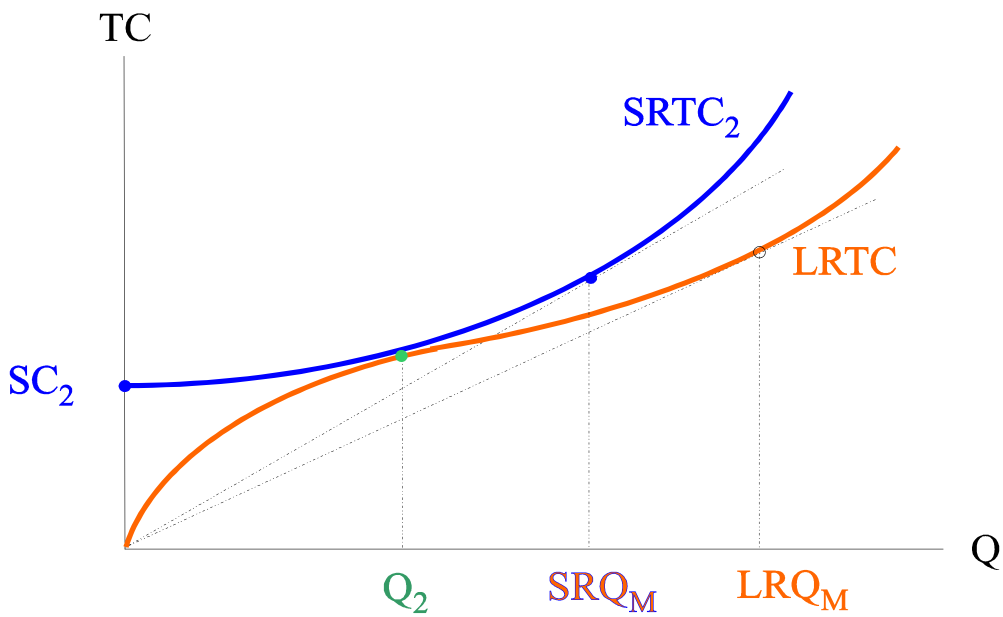
cost-minimizing input choices are given by

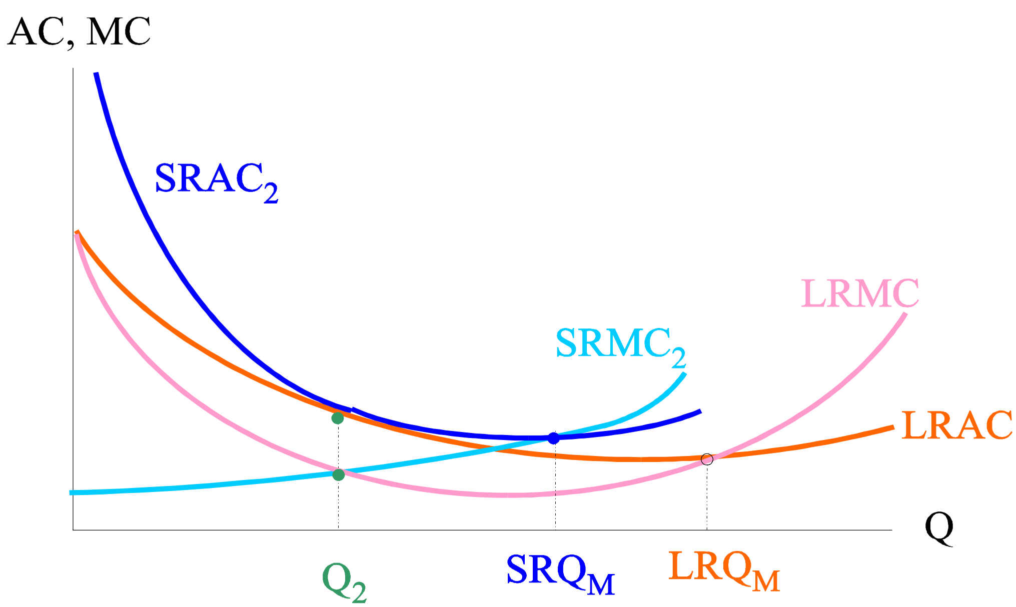
$$L^* = \partial C/\partial w, \quad K^* = \partial C/\partial r$$



SHORT- AND LONG-RUN COST CURVES







Q = quantity, TC = total cost, AC = average cost, MC = marginal cost

LR = long run, SR = short run, SC = sunk cost

Subscript 2 denotes that short run in which K is optimal to produce Q_2

Subscript M stands for the AC-minimizing quantity

$SRQ_M > Q_2$; even in SR, there may be some scale economies beyond Q_2