UNCERTAINTY – ALTERNATIVE THEORIES

Behavior inconsistent with expected utility theory:
Some examples

1. Allais Paradox
Consider the four lotteries below
Column headings are prizes; cell entries probabilities

Lottery	\$ 0	\$ 1,000	\$ 5,000
A	0	1	0
B	0.1	0.8	0.1
C	0.90	0.10	0
D	0.91	0.08	0.01

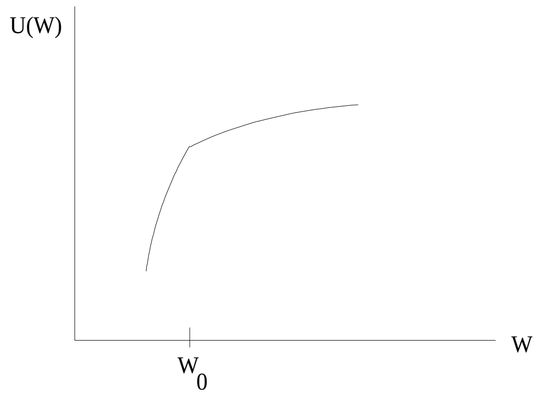
Write Z for the sure prospect of 0. Then

$$C = A \text{ with prob. } 0.1 + Z \text{ with prob. } 0.9$$
$$D = B \text{ with prob. } 0.1 + Z \text{ with prob. } 0.9$$

By the independence axiom,
an expected utility maximizer who
prefers A to B should also prefer C to D, and
if he prefers B to A, should also prefer D to C

But many people choose A over B, and D over C

2. Kinked Utility



vN-M type utility but with kink at initial wealth
Values losses from status quo much more than gains

3. Minimizing maximum regret
When comparing choices A and B, regret of A is

$$\max_{i \,\in\, \text{Scenarios}} \; \max(\, U_i(B) - U_i(A), 0\,)$$

4. Errors in calculating probabilities
Treating small probability events as if impossible
Not applying correct Bayes' rule when
updating probabilities given some information

Expected utility approach still dominates
in most applications – finance, game theory etc.
But alternatives being explored at research level

DEMAND FOR INSURANCE

Loss L in scenario 2 (prob. π_2) Endowments $(W_0, W_0 - L)$
Each dollar of coverage requires insurance premium p

If insurance is actuarially (statistically) fair, $p = \pi_2$.

If buy X dollars of coverage, final wealths

$$W_1 = W_0 - p\,X, \quad W_2 = W_0 - L - p\,X + X$$

Choose X to maximize

$$EU(W) = \pi_1\,U(W_0 - p\,X) + \pi_2\,U(W_0 - L + (1-p)\,X)$$

FONC

$$-p\,\pi_1\,U'(W_0 - p\,X) + (1-p)\,\pi_2\,U'(W_0 - L + (1-p)\,X) = 0$$

SOSC is $U'' < 0$, risk-aversion.
If fair insurance ($p = \pi_2$, $1 - p = \pi_1$) FONC becomes:

$$U'(W_0 - p\,X) = U'(W_0 - L + (1-p)\,X), \qquad W_1 = W_2$$

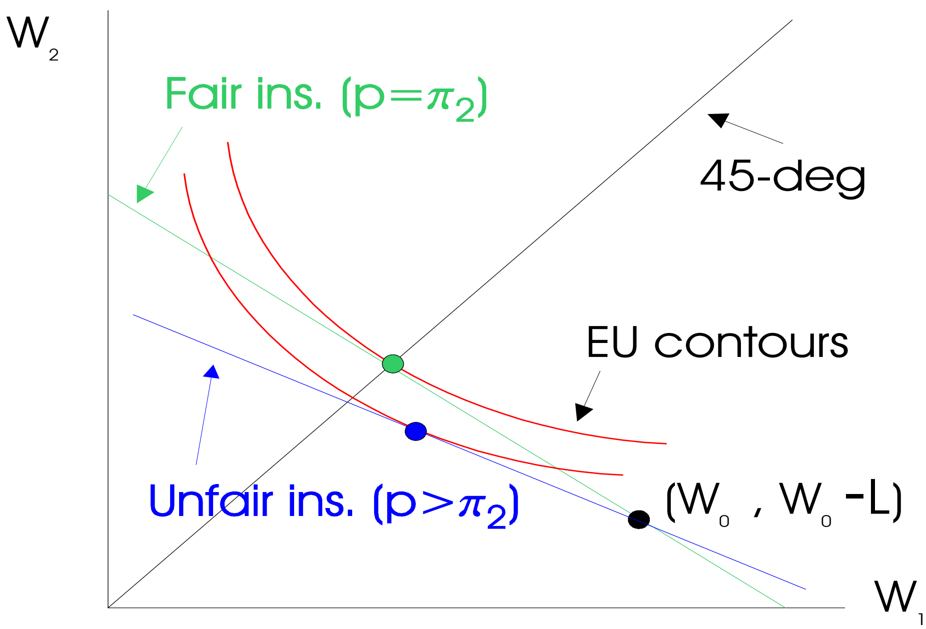
so risk is eliminated – full coverage $X = L$ optimum
Ins. Co.'s expected profit $E\Pi = p\,X - \pi_2\,X$, so
fair insurance will be available if:

(1) Risk-neutral insurers (by law of large nos ?)
(2) Perfect competition among insurers $\Rightarrow E\Pi = 0$
(2) No (minimal) admin. costs, no info. asymmetry

Alternative view: eliminate X from W_1, W_2 equations:

$$(1-p)\ W_1 + p\ W_2 = (1-p)\ W_0 + p\ (W_0 - L)$$

Budget constraint for "contingent claims to dollars"
Prices p, $(1-p)$. Slope $= (1-p)/p$
Subject to this, max $EU = \pi_1\ U(W_1) + \pi_2\ U(W_2)$
Probabilities must be exogenous for symmetric info.



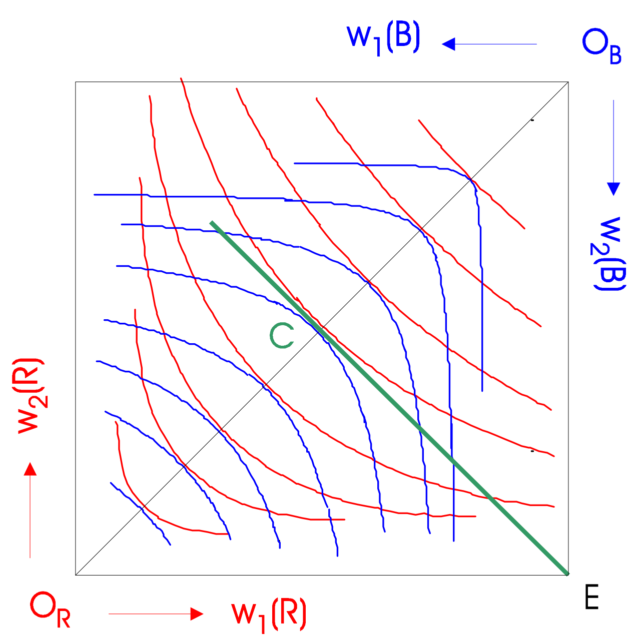
If insurance is fair, $p = \pi_2$
slope of budget line = 45-degree-line-MRS
so tangency (optimum) at 45-degree-line

If "unfair" (loaded) insurance, $p > \pi_2$
slope of budget line < 45-degree-line-MRS
Less than full insurance is optimal

TRADING RISK IN MARKETS

Markets held before uncertainty is resolved
Buy/sell "contingent claims", like betting slips
Simplest of these – Arrow-Debreu Securities (ADS)
    Basic or elementary scenarios $i = 1, 2, \ldots n$
    ADS_i is claim to \$1 if scenario i, nothing otherwise
    Prices p_i paid in advance
If money can be stored between now and time when
    uncertainty resolved and claims settled, $\sum_i p_i = 1$
If sure int. rate r between now and then, $= 1/(1+r)$
Equilibrium prices p_i depend on probabilities π_i
    and on extent of, and attitudes toward, the risks
EXAMPLE 1 – No aggregate risk
  Individual risk can be fully insured by trade at fair prices

Two scenarios
Total W same in both
Objective Probs. (1/2 each)
B more risk-averse than R
But MRS on 45-degree
 $\pi_1/\pi_2 = 1$ for both
So eqm. on that line
 $p_1/p_2 = \pi_1/\pi_2$

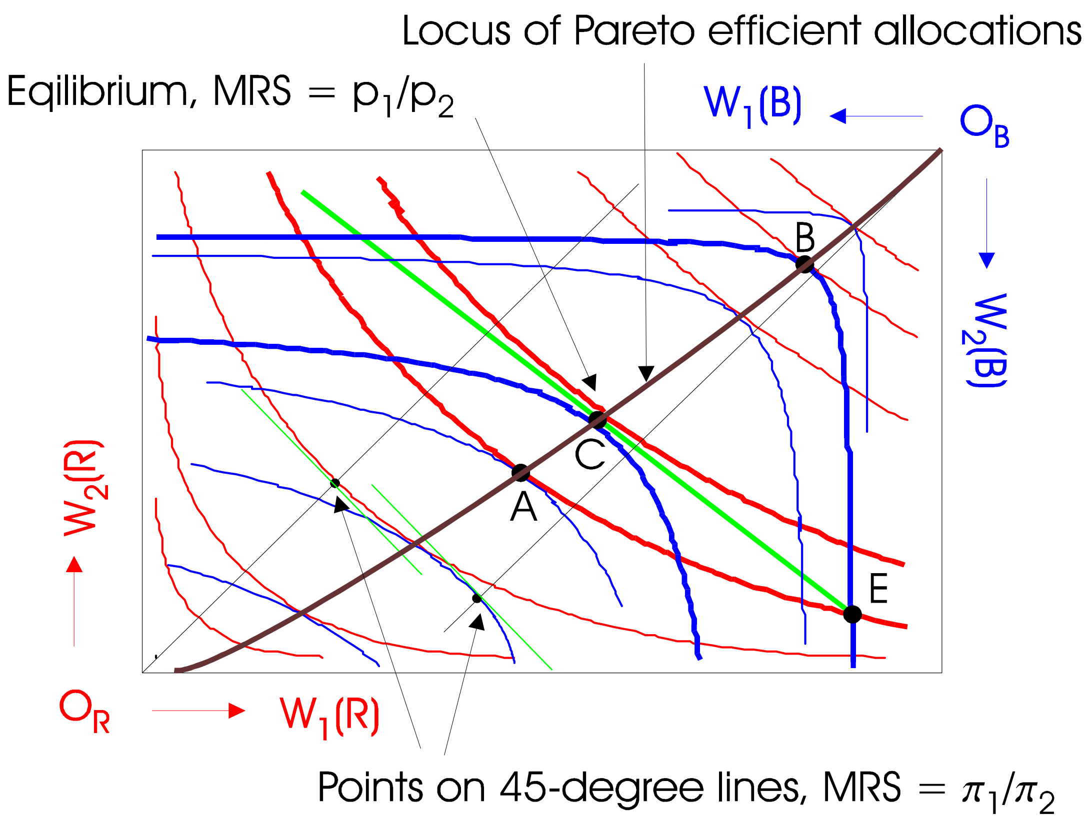


EXAMPLE 2 – Aggregate risk

Total $W_1 > W_2$: Scenario 1 “good”, 2 “bad”

Objective probabilities π_1, π_2

E = initial endowment, AB = core, C = equilibrium



Pareto efficiency, Core, Equil’m as in GE Theory (Wk.7)

ADS’s achive efficient allocation of risk !

B is more risk-averse than R – so efficient points are relatively closer to B’s 45-degree line

At any efficient risk-allocation, $p_1/p_2 < \pi_1/\pi_2$

Difference depends on risk-aversions of traders

If one is risk-neutral, $p_1/p_2 = \pi_1/\pi_2$