ECO 305 – Fall 2003

Precept Week 9 – Question

Strategic Commitment in Oligopoly

In quantity-setting duopoly, a firm will make more profit if it can seize the first move (become a Stackelberg leader) than in the Cournot equilibrium where choices are simultaneous. Why doesn't a firm simply tell the other in advance that it is determined to produce its Stackelberg leadership output? The problem is that such verbal threats are not automatically credible because there is no genuine incentive to go through with them. Here we look at an indirect method by which a threat to produce more than one's Cournot output can be made credible.

We consider a homogeneous-product Cournot duopoly with a twist: one firm is given the opportunity to undertake a costly action z that will reduce its marginal cost. This could be R-and-D, or investment.

Denoting the common price by p and the quantities by x_1, x_2, the demand function is

$$p = 9 - (x_1 + x_2)\,.$$

The marginal cost of firm 2 is constant: $c_2 = 3$. That of firm 1 is given by

$$c_1 = 3\,(1 - z)\,,$$

so z is the proportional reduction in marginal cost, which is achieved by undertaking a fixed and sunk cost $12\,z^2$. (The numbers are rigged to produce simple answers, but the principle is more general as we shall see.)

(a) Output-Only Game – base case

Before we begin the analysis, consider for later comparisons a base case where firm 1 does not have this choice, that is, $z \equiv 0$. Find the Cournot equilibrium. Show that:

$$x_1 = x_2 = 2, \qquad p = 5, \qquad \Pi_1 = \Pi_2 = 4\,.$$

where Π_1, Π_2 denote the profits of the two firms.

(b) Single-Stage Game

Now suppose all action takes place simultaneously. Firm 1 has two choice variables, x_1 and z, whereas firm 2 has only one choice variable, x_1. We can find a Nash equilibrium of this game.

Express the profit of firm 1 (Π_1) as a function of (x_1, x_2, z). Write down the FONCs for (x_1, z) to maximize Π_1 for given x_2.

Express the profit of firm 2 (Π_2) as a function of (x_1, x_2). Write down the FONC for x_2 to maximize Π_2 for given x_2.

You may want to check the SOSCs but they are OK in this example.

Solve the three FONCs for (x_1, x_2, z). That is the Nash equilibrium. Show that the solution is

$$z = 1/3, \quad x_1 = 8/3 = 2.67, \qquad x_2 = 5/3 = 1.67, \qquad p = 14/3 = 4.67$$

and the profits are

$$\Pi_1 = 52/9 = 5.8, \qquad \Pi_2 = 25/9 = 2.8$$

Compare these magnitudes to the corresponding ones in the base case. Explain the economic intuition for the differences.

(c) Two-Stage Game

Now suppose the actions takes place in two stages. First, firm 1 chooses z, and firm 2 gets to see this choice of z. Then the two firms choose their outputs x_1, x_2 respectively, resulting in their Cournot duopoly outcome.

In this situation, firm 1 in its choice of z will look ahead to the effect this will have on the duopoly equilibrium. So you need to solve for the rollback or subgame-perfect equilibrium.

To calculate the Cournot equilibrium of the second stage (firm 2's look-ahead calculation), fix z as a general algebraic constant (parameter). Take the expressions for Π_1 and Π_2 above, and two FONCs: x_1 maximizes Π_1 for given x_2, and x_2 maximizes Π_2 for given x_1. (All the time, z is held fixed as a parameter.) Solve these FONCs for x_1 and x_2 in terms of z. Show that the result is

$$x_1 = 2 + 2\ z, \qquad x_2 = 2 - z$$

Now substitute these functions into the expression for Π_1. That is the profit that firm 1 calculates it will make by choosing z, taking into account the implications for the Cournot equilibrium values of x_1 and x_2 at the second stage. It remains to choose z to maximize this Π_1. Show that the outcome is

$$z = 1/2$$

Then in the resulting equilibrium

$$x_1 = 3, \qquad x_2 = 1.5, \qquad p = 14/3 = 4.5$$

and the profits are

$$\Pi_1 = 6, \qquad \Pi_2 = 2.25$$

Compare this to the single-stage game above. How do the various magnitudes differ? What is the economic intuition for these differences?

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Precept Week 9 – Solutions
Strategic Commitment in Oligopoly

Remember that the demand function is

$$p = 9 - (x_1 + x_2)\,.$$

The marginal cost of firm 2 is constant: $c_2 = 3$. That of firm 1 is given by

$$c_1 = 3\,(1 - z)\,,$$

so z is the proportional reduction in marginal cost, which is achieved by undertaking a fixed and sunk cost $12\ z^2$.

(a) Output-Only Game – base case

Here $z \equiv 0$, so the constant marginal costs are 3 each, and the two firms's profits are

$$\Pi_1 = [9 - (x_1 + x_2) - 3]\ x_1, \qquad \Pi_2 = [9 - (x_1 + x_2) - 3]\ x_2$$

The Cournot-Nash FONCs (best response functions) are

$$\begin{aligned} \frac{\partial \Pi_1}{\partial x_1} &\equiv 6 - 2\,x_1 - x_2 = 0 \\ \frac{\partial \Pi_2}{\partial x_2} &\equiv 6 - x_1 - 2\,x_2 = 0 \end{aligned}$$

The solutions to these are easily seen to be

$$x_1 = x_2 = 2, \qquad \text{and then} \qquad p = 5, \qquad \Pi_1 = \Pi_2 = 4\,.$$

Single-Stage Game

First suppose all action takes place simultaneously. Firm 1 has two choice variables, x_1 and z, whereas firm 2 has only one choice variable, x_1. We can find a Nash equilibrium of this game.

The profit of firm 1 is

$$\begin{aligned} \Pi_1 &= \{\ [9 - (x_1 + x_2)] - 3\,(1 - z)\ \}\ x_1 - 12\ z^2 \\ &= \{\ 6 + 3\,z - x_1 - x_2\ \}\ x_1 - 12\ z^2 \end{aligned} \qquad (1)$$

Firm 1 takes x_2 as given, and chooses x_1 and z to maximize Π_1. The FONCs for this are

$$\begin{aligned} \partial \Pi_1 / \partial x_1 &\equiv 6 + 3\,z - 2\ x_1 - x_2 = 0 \\ \partial \Pi_1 / \partial z &\equiv 3\ x_1 - 24\ z = 0 \end{aligned} \qquad (2)$$

The profit of firm 2 is

$$\Pi_2 = [\,9 - (x_1 + x_2) - 3\,]\ x_2 = (6 - x_1 - x_2)\ x_2\,,$$

The FONC for its choice of x_2 is

$$\partial \Pi_2 / \partial x_2 \equiv 6 - x_1 - 2\,x_2 = 0.$$

To find the Nash equilibrium we solve all three FONCs jointly. The result is

$$z = 1/3, \quad x_1 = 8/3 = 2.67, \qquad x_2 = 5/3 = 1.67, \qquad p = 14/3 = 4.67$$

and the profits are

$$\Pi_1 = 64/9 - 12/9 = 52/9 = 5.8, \qquad \Pi_2 = 25/9 = 2.8$$

Compared to the base case: (1) firm 1's output has gone up, (2) firm 2's output has gone down but by less than firm 1's goes up, (3) the price has gone down, (4) firm 1's profit has gone up and firm 2's profit has gone down, (5) the total profit has gone up.

This can be understood as follows. Firm 1's action has some genuine merit because it lowers the cost of production. But the action has another effect – it changes the "balance of power" in the duopoly in favor of firm 1. Write firm 1's best response function explicitly solved for x_1 in terms of x_2 and z, namely

$$x_1 = (6 - x_2 + 3\,z)\,/\,2 \tag{3}$$

Compare the two Cournot (quantity-setting) duopoly games for two separate and fixed values of z, namely $z = 0$ and $z = 1/3$. In going from $z = 0$ to $z = 1/3$, firm 1's best response function shifts to the right. Then the Cournot equilibrium slides down firm 2's best response function

$$x_2 = (6 - x_1)\,/\,2\,. \tag{4}$$

So firm 1's output goes up and that of firm 2 goes down. The figure shows this.

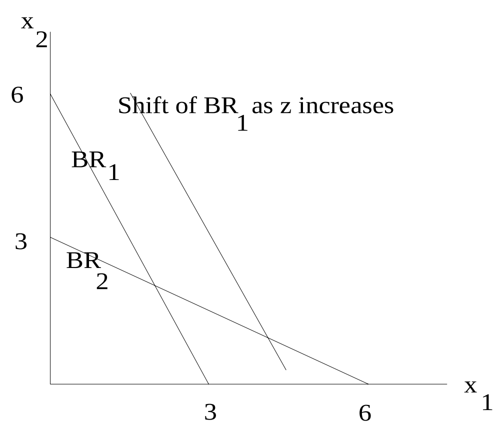

Figure 1: Shift of firm 1's best response function as z increases

In the two-stage game to follow, firm 1 makes its choice of z in stage 1 to take strategic advantage of just this fact.

Two-Stage Game

Now suppose the actions takes place in two stages. First, firm 1 chooses z, and firm 2 gets to see this choice of z. Then the two firms choose their outputs x_1, x_2 respectively, resulting in their Cournot duopoly outcome.

In this situation, firm 1 in its choice of z will look ahead to the effect this will have on the duopoly equilibrium. So let us solve out the duopoly equilibrium taking z as fixed at stage 1. Now the profits of the two firms are

$$\begin{aligned}\Pi_1 &= \{\,6 + 3\,z - x_1 - x_2\,\}\,x_1 - 12\,z^2 \\ \Pi_2 &= \{\,6 - x_1 - x_2\,\}\,x_2\end{aligned}$$

So their respective FONCs are

$$\begin{aligned}\partial\Pi_1/\partial x_1 &\equiv 6 + 3\,z - 2\,x_1 - x_2 = 0 \\ \partial\Pi_2/\partial x_2 &\equiv 6 - x_1 - 2\,x_2 = 0.\end{aligned}$$

Solving these for x_1 and x_2 yields

$$x_1 = 2 + 2\ z, \qquad x_2 = 2 - z\,. \tag{5}$$

Then

$$p = 5 - z$$

and

$$\begin{aligned} \Pi_1 &= [\,(5 - z) - 3\,(1 - z)\,]\ (2 + 2\,z) - 12\ z^2 \\ &= 4\ (1 + z)^2 - 12\ z^2 \end{aligned}$$

This is the profit that firm 1 when choosing z at stage 1 rationally calculates it will get. The FONC for z to maximize this is

$$d\Pi_1\,/\,dz \equiv 8\,(1 + z) - 24\,z = 8 - 16\,z = 0\,. \tag{6}$$

The SOSC is obviously satisfied, and the optimum is

$$z = 1/2$$

Then in the resulting equilibrium

$$x_1 = 3, \qquad x_2 = 1.5, \qquad p = 14/3 = 4.5$$

and the profits are

$$\Pi_1 = 6, \qquad \Pi_2 = 2.25$$

Compare this to the single-stage game above. In the two-stage game firm 1 reduces its marginal cost even more than it did in the one-stage game (which cost was already less than that in the output-choice-only base case). So firm 1's output is even higher, and that of firm 2 is even lower. Total output goes up further, so price goes down further. Firm 1's profit increases further, and firm 2's profit falls even farther. Total profit is lower in the two-stage game than in the one-stage game. Total profit remains higher in the two-stage game than in the output-only base case, but this is not a general result and in other examples the comparison could go the other way.

Firm 1's new ability to choose z first, so firm 2 can observe it and react, gives firm 1 an extra strategic advantage. It commits to an additional reduction in its marginal cost, so that firm 2 will see this and recognize that in equilibrium it should be producing even less. Since a lower output by firm 2 means a more favorable demand situation for firm 1, it is able to expand output to take advantage of this and make more profit.

A more mathematical way to see this is to compare the z-FONCs (6) and (2) for the one-stage and two-stage games. The two-stage choice takes into account the second-stage reaction of x_1 and x_2 knowing the previously chosen z, while everything being simultaneous in the one-stage choice this reaction is not taken into account. Take the expression (1) for Π_1 and differentiate it using (5) and the chain rule:

$$\begin{aligned} \left.\frac{\partial \Pi_1}{\partial z}\right|_{\text{two-stage}} &= \left.\frac{\partial \Pi_1}{\partial z}\right|_{\text{one-stage}} + \frac{\partial \Pi_1}{\partial x_1}\,\frac{\partial x_1}{\partial z} + \frac{\partial \Pi_1}{\partial x_2}\,\frac{\partial x_2}{\partial z} \\ &= [3\,x_1 - 24\,z] + 0 + (-x_1)\,(-1) \\ &= 3\,(2 + 2z) - 24\,z + 0 + (2 + 2z) = 8 - 16\,z \end{aligned}$$

This is the same as the FONC (6) of the two-stage case, but the derivation gives further insight. The first term on the right hand side is just the FONC in the one-stage game. The second term is zero because of the x_1-FONC of the second-stage quantity game. The third term is what gives the added strategic advantage – an increase in z lowers x_2 and this increases Π_1. Thus the strategic advantage of choosing z first consists of the effect it has on the *other* firm's subsequent quantity choice.

There are two crucial things in this. [1] An increase in firm 1's output shifts down the *marginal profit* of firm 2:

$$\partial \Pi_2 / \partial x_2 = 6 - x_1 - 2\ x_2$$

decreases as x_1 increases for any given x_2. (The technical term for this is that firm 1's output x_1 is a *strategic substitute* for firm 2's output.) That is why firm 2's best response function (equation (4)) is downward-sloping. Therefore, if firm 1 can credibly commit itself to acting more aggressively, that is, producing a higher x_1, it can induce firm 2 to back off, that is, produce a smaller x_2. This works to firm 1's advantage. [2] Firm 1's costly action of increasing z is a way to credibly commit itself to acting more aggressively. In firm 1's best response function (equation (3)), x_1 increases as z increases, for any fixed x_2. That is, the function shifts to the right, as in Figure 1. The best-response function is an objective fact; it is credible, unlike a mere verbal threat "I will produce more" which firm 1 would not have any incentive to carry out (that would not be its best response).

To sum up, here we have a situation in which (1) the firms' outputs are strategic substitutes, so firm 1 benefits by producing more (competing more aggressively), and (2) stage 1 investment (or R-and-D) provides firm 1 a way to be credibly more aggressive. That is why firm 1 makes a larger investment when it is able to make it at stage 1. (Investment made at stage 2, that is, simultaneously as firm 2 is choosing its own output, would not have this commitment value of a *fait accompli.*)

In other applications of this general idea of two-stage games, firms' actions may be strategic complements rather than substitutes, and stage 1 action may make firm 1 more or less aggressive. Fudenberg and Tirole (American Economic Review, May 1984) have given us a memorable taxonomy of the possibilities, shown in the table below. Ours was an example of the "Top-Dog" strategy.

Table 1: The Fudenberg-Tirole Zoo

		Investment makes player 1	
		aggressive	weak
Slope of Player 2's reaction function	Down (strategic substitutes)	Top Dog: Overinvest to become more aggressive	Lean and Hungry: Underinvest to become more aggressive
(Relationship between the two strategies)	Up (strategic complements)	Puppy Dog: Underinvest to become less aggressive	Fat Cat: Overinvest to become less aggressive

Many concepts in industrial organization and international trade (for example "strategic trade policy") are applications of this. Related references: Bulow, Geanakoplos and Klemperer (Journal of Political Economy 1985), Eaton and Grossman (Quarterly Journal of Economics 1986).

In this example we gave firm 1 the ability to choose this strategic variable z simply by assumption. If both firms can choose such a variable, z_1 for firm 1 and z_2 for firm 2, then the two-stage game will have to be solved for a subgame-perfect Nash equilibrium: First solve the second-stage Cournot-Nash equilibrium in (x_1, x_2) for given (z_1, z_2). Substitute the solutions,namely (x_1, x_2) as functions of (z_1, z_2), into the profit expressions to express them as functions of (z_1, z_2) alone (this embodies the rational look-ahead of the firms). Then solve for the Nash equilibrium of (z_1, z_2) using these profit functions. In such a game, the two firms will be competing with their choices of z_1 and z_2 respectively, in an attempt to force the other firm to lower its x_2 and x_1 respectively. This game itself becomes a Prisoner's Dilemma.

Purely optional extra – For those of you who are interested in pursuing this line of theory further, a more general mathematical theory of two-stage games is available on the web site.