ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 1 – Due September 25 in class

Question 1: (45 points)

Note: Your graph in part (a) should suggest how to proceed in parts (b)-(e), but you must check the appropriate calculus conditions for your answers to those parts.

Consider the real-valued function f defined over the interval [-3,3] by

$$f(x) = \begin{cases} -4\,x - 3\,x^2 - \frac{2}{3}\,x^3 & \text{for } -3 \leq x < 0 \\ 0 & \text{for } x = 0 \\ 2\,x - \frac{3}{2}\,x^2 + \frac{1}{3}\,x^3 & \text{for } 0 < x \leq 3 \end{cases}$$

(a) Sketch a rough graph of the function. Use a calculator or a computer program such as Mathematica if you can; else calculate a few values by hand. Integer and half-integer values of x will suffice to give you a good idea.

(b) Find all critical points of the function. Identify the local maxima and minima.

(c) Is the function non-differentiable anywhere? Is there a local maximum or minimum at this point?

(d) Does the function have any local maxima or minima at its end-points?

(e) Find its global maximum and minimum.

Question 2: (35 points)

In all three parts that follow, x and y are restricted to be non-negative real numbers.

(a) Maximize $3\,x + 4\,y$ subject to $x^2 + y^2 = 25$.

(b) Minimize $x^2 + y^2$ subject to $3\,x + 4\,y = 50$.

(c) Maximize $x^2 + y^2$ subject to $3\,x + 4\,y = 50$.

(Hint: In each case, first draw rough sketches of the constraint curve and one or two level curves of the objective function. This will tell you whether Lagrange's Method can be applied.)

Question 3: (20 points)

You have two final exams upcoming, and have to decide how to allocate your time during the reading period. After eating, sleeping, exercising, and maintaining some human contact, you will have 15 hours each day in which to study for your exams. You have figured out that your grade point average (G) from your two courses, Mathematical Methods and French Fiction, takes the form

$$G = \frac{4}{5\sqrt{3}} \left[\sqrt{F} + 2\,\sqrt{M} \right] ,$$

where F is the number of hours per day spent studying for French Fiction and M is the number of hours per day spent studying for Mathematical Methods (these are to be regarded as continuous variables). You only care about your GPA. What is your optimal allocation of study time? If you follow this optimal strategy, what will be your GPA? What will be the shadow value, measured in GPA units, of the study time?