ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 2 – Due October 2 in class

Question 1 (20 points):

You have just emerged from medical school with a debt service burden of $25,000 per year, and have set up practice. You have to decide how hard to work. For each hour of work, you expect to earn $50 (after subtracting expenses of maintaining your office, taxes, etc.). Your utility function for a full year is

$$U(I, H) = \ln(I) + 2\ \ln(5000 - H)\,,$$

where H is the number of hours you work during the year, and I is what is left of your annual income after expenses, taxes, and debt service.

(a) What is your budget constraint linking I and H?

(b) Find your optimal number of hours of work.

(c) If taxes go up so you are left with only $40 per hour of work, will you work more or fewer hours? Explain the economic intuition for the result.

Question 2 (50 points):

There are two goods, whose quantities are denoted by X and Y, each being a real number. An individual's consumption set consists of all (X, Y) such that $X \geq 0$ and $Y > 1$. His utility function is:

$$U(X, Y) = 4\ \ln(X + 2) + \ln(Y - 1)\,.$$

The price of X is p and that of Y is q; total income is I, The aim of the question is to find the consumer's demand functions and examine their properties. You need not worry about second-order conditions. Proceed as follows:

(a) First solve the problem by Lagrange's method, ignoring the constraints $X \geq 0, Y > 1$. Show that the solutions for X and Y that you obtain are valid demand functions if and only if $I \geq \frac{1}{2}\,p + q$.

(b) Next suppose $I \leq \frac{1}{2}\,p + q$. Solve the utility maximization problem subject to the budget constraint and an additional constraint $X \geq 0$, using Kuhn-Tucker theory. Show that the solutions for X and Y you get here are valid demand functions if and only if $q < I \leq \frac{1}{2}\,p + q$. What happens if $I \leq q$?

In each of the following parts, consider the above cases (a) and (b) separately.

(c) Show that the demands are homogeneous of degree 0 in (p, q, I) jointly.

(d) Find the algebraic expressions for the income elasticities of demand for X, Y. Which, if either, of the goods is a luxury?

(e) Find the marginal propensities to spend on the two goods. Which, if either, of the goods is inferior?

(f) Find the algebraic expressions for the own price derivatives $\partial X/\partial p$, $\partial Y/\partial q$. Which, if either, of the goods is a Giffen good?

Question 3: (30 points)

(In this question, you can use Lagrange's method taking for granted that the second-order conditions are satisfied and boundary solutions do not arise.)

(a) There are two goods X and Y, with prices p and q. A consumer's utility function is

$$U(X, Y) = X^{1/4}\, Y^{3/4}\,.$$

(a) Find algebraic expressions for the quantities that solve the usual problem

$$\text{maximize } U(X, Y) \text{ subject to } p\,X + q\,Y \leq I\,.$$

These are functions of (p, q, I), and are called Marshallian demand functions. Denote them by X^m and Y^m. Find the algebraic expression for the resulting utility u also as a function of (p, q, I).

(b) Now consider the mirror-image problem: how much income is needed to achieve at least a specified target utility level u if the consumer makes the most economical choices:

$$\text{minimize } p\,X + q\,Y \text{ subject to } U(X, Y) \geq u\,.$$

These are functions of (p, q, u), and are called Hicksian demand functions. Denote them by X^h and Y^h.

(c) Evaluate $\partial X^h / \partial q$ and $Y^m\ \partial X^m / \partial I$. Show that the two are equal when u and I are related by the expression you found in (a) above.