ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 3 – Due October 9 in class

Question 1: (40 points)

Throughout this question, there are two goods X and Y, whose respective prices should be denoted by P and R.

(a) Find the expenditure function for each of the following two utility functions. Assume that the second-order conditions comparing the convexity of objective contours and the constraint are satisfied, and use Lagrange's method. Hint: Transform the constraint for ease of differentiation using the method developed in the Precept Handout for Week 3, Solution to Question 3.

(1) $U(X,Y) = [\ X^{-1} + Y^{-1}\ ]^{-1}$
(2) $U(X,Y) = [\ X^{1/2} + Y^{1/2}\ ]^{2}$

(b) Find the expenditure function for each of the following two utility functions. Note: Neither has a "regular" maximum found using Lagrange's method. Base your solutions purely on sketches of the indifference maps and the budget constraint.

(1) $U(X,Y) = X + Y$ (case where X and Y are perfect substitutes one-for-one)
(2) $U(X,Y) = \min(X,Y)$ (case of zero substitution, sometimes called "perfect complementarity")

Question 2: (30 points)

Notes: [1] In this question, you can use the standard results on demands for Cobb-Douglas utility functions without deriving them from first principles. [2] The budget constraint is similar to the one we derived in the precept discussions of week 3. [3] There may be local maxima that are not global. As usual, drawing a rough picture of the budget constraint may help your thinking.

Consider two undergraduates, Fritz Fresser and Gaby Geplauder. Each has a monthly allowance of \$100, which can be spent on two things: pizza, and talking to friends back home on the phone. Each pizza costs \$10. The phone company offers two plans. The first has no fixed fee, and charges 10 cents per minute of use. The second has a fixed fee of \$40 per month, plus $3\frac{1}{3}$ cents per minute of use.

Fritz has the utility function

$$U_F = (P_F)^4\ M_F\ ,$$

where P_F is the number of pizzas he consumes each month, and M_F is his phone usage measured in units of "hundred minutes". With similar notation, Gaby's utility function is

$$U_G = P_G\ M_G\ .$$

Find the optimal choice – the choice of phone plan, the number of minutes spent on the phone, and the number of pizzas eaten – for each.

Question 3: (30 points)

In the 1970s, gasoline was rationed in many countries. In this question you are asked to analyze such a situation.

Della Driver's utility is a function $U(G, R)$, where G denotes her consumption of gallons of gasoline per week, and R stands for her consumption of all other goods in the economy, measured simply by the amount spent (measured in £, the currency unit) on these other goods.

$$U(G, R) = 0.05 \ \ln(G) + 0.95 \ \ln(R) \, .$$

Della's income is £1000 per week. The price of gasoline is £1 per gallon.

(a) Della is not allowed to buy any more than 20 gallons of gasoline for the whole week. If she buys the maximum allowed, what will be the values of G and R? What is her marginal rate of substitution (the numerical value of dR/dG along her indifference curve) at this point? Is it greater or less than the price?

(b) Show this in a figure and hence answer whether it is optimal for Della to buy the maximum amount of gasoline allowed.

(c) Next suppose a black market in gasoline opens up, where she can buy additional quantities for £1.40 per gallon (while continuing to buy the ration of 20 gallons at the "legal" price of £1). How much gasoline will she buy on the black market? (Hint: You need to be careful in setting up the budget constraint for this case.)