ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 4 – Due October 16 in class

Question 1: (40 points)

There are two goods, food and clothing, whose quantities are to be denoted by F and C and prices by P_F and P_C respectively. There is a consumer whose income is to be denoted by M and utility by U. His expenditure function is known to be

$$M = M^*(P_F, P_C, U) = (P_F)^{1/2} \ (P_C)^{1/2} \ U \ .$$

(a) Write down his Marshallian or uncompensated demand functions. Do not derive them from first principles, just state them using standard Cobb-Douglas results.

(b) Initially $M = 100$, $P_F = 1$, and $P_C = 1$. What quantities does the consumer buy, and what is his resulting utility?

(c) Now the price of food rises to $P_F = 1.21$, while income and the price of clothing are as before. What quantities does the consumer buy and what is his resulting utility?

(d) Suppose the increase in the price of food was caused by the government levying a tax of 21 percent on food. What is the government's revenue from this tax?

(e) If the government wants to compensate the consumer by giving him some extra income, how much extra income would be needed to restore him to the old utility level? Is the government's revenue from the tax on food itself sufficient to provide this compensation? What is the economic intuition for your answer?

(f) If the government tries to compensate the consumer by giving him enough extra income to enable him to purchase the same quantities as he did at the original income and prices of part (b), how much extra income would the government have to give him? With this income and the new prices, what quantities will the consumer actually buy? What will be his resulting utility?

(g) In a graph with F on the horizontal axis and P_F on the vertical axis, show the consumer's choices with the income $M = 100$ and the two prices $P_F = 1$ and $P_F = 1.21$. Calculate the dead-weight loss caused by the tax on food as the area of a triangle, as you would in ECO 102. What is the correct dead-weight loss and how does your area compare with it? What are the reasons for the difference between the two measures?

Question 2: (60 points)

A firm can hire capital at a rental price r and labor at a wage w. To produce anything at all requires one unit of capital; this is fixed and sunk in the short run, and fixed but not sunk in the long run. If in a unit of time the firm employs L units of labor, and rents K units of capital (in addition to the one unit needed as the sunk and/or fixed cost), its output Q is given by the production function

$$Q = K^{1/3} \ L^{1/6} \ .$$

(a) In the short run, the firm is committed to hire a fixed amount of capital $1 + K$, and can vary its output Q only by employing an appropriate amount of labor L. Find algebraic expressions for the firm's short-run total, average, and marginal cost functions.

(b) In the long run, the firm can vary both capital and labor. Find algebraic expressions for the firm's long-run total, average, and marginal cost functions.

(c) To link the short-run and the long-run cost curves, take the short-run average cost curve, and for given Q, find the K (as a function of Q) that minimizes short-run average cost. Substitute this in the short-run average cost function, reducing it to a function of Q, r and w. Verify that it is the same as the long-run average cost function.

(d) Show that the quantity that minimizes the firm's long-run average cost is given by

$$Q = \frac{2^{1/3}}{3^{1/2}}\, r^{1/6}\, w^{-1/6}\,.$$

Observe that this Q is an increasing function of r. What is the economic intuition for this finding?