ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 5 – Due November 13 in class

Question 1: (50 points)

In this question, the graphs you are asked to draw don't have to be very accurate, but they should represent the magnitudes reasonably clearly.

In this problem we will consider the fish industry on the island of San Serife. For this purpose we will aggregate all the other goods into one composite, and measure it in units of the island's currency Arial. Thus the price of the other good is 1.

(a) There are 160 consumers, each with the utility function

$$U(q, y) = y + 10\ q - 5\ q^2\ ,$$

where q is the consumption of fish and y the consumption of the other good. Let p denote the price of fish. Find one consumer's demand function (q expressed as function of p). Be careful about boundary solutions with $q = 0$, but ignore boundary solutions with $y = 0$. Find the market demand function. Show this in a graph with p on the vertical axis and the market quantity Q on the horizontal axis.

(b) The fishing industry consists of several firms. Each firm, to produce and sell anything at all, must get a boat and hire a market stall. The cost per period of owning a boat (the interest on the money tied up) is 3 Arials, and the cost of hiring a market stall is 1 Arial. In the long run, both of these costs are fixed but avoidable. In the short run, the cost of the boat is (fixed and) sunk (The cost is sunk, not the boat!) because boats have no alternative use, whereas the cost of the market stall is (fixed but) avoidable because the same stall can be used to sell other things. The variable cost of producing output q is q^2.

Write down expressions for each firm's long run total cost (LRTC), long run average cost (LRAC), short run total avoidable cost (SRTAC), short run average avoidable cost (SRAAC), and marginal cost (MC), in each case as functions of q. Find the values of q that minimize LRAC and SRAAC, and the minimum values of these two average costs. Find the equation for the firm's short run supply curve. Draw rough sketches of the average and marginal costs and the supply curve.

(c) In the long run, there is free entry and exit of fishing firms. What is the industry's long run supply curve?

(d) Suppose the industry is initially in long run equilibrium. Putting together the market demand curve you found in part (a) and the industry supply curve you found in part (c), find the long run equilibrium price. How many firms operate in this equilibrium? What is the profit of each? What is the total of all consumers' surpluses?

(e) Now suppose the government levies a tax of 2.5 Arials per unit of fish. Find the new short run equilibrium with the same number of firms as in the original long run equilibrium. What is the price paid by the consumers? What is the price received by the firms? What is the profit of each firm? Will any firms want to exit in the short run? What is the aggregate

loss of consumer surplus? How much tax revenue does the government collect? What is the dead-weight loss?

(f) Will firms want to exit in the long run with the tax? In this new long run equilibrium, what is the price paid by the consumers? What is the price received by the firms? How many firms are active? What is the profit of each firm? What is the aggregate loss of consumer surplus? How much tax revenue does the government collect? What is the dead-weight loss?

Question 2: (50 points)

John has an initial endowment of 10 kilos of beef and 8 litres of wine. Marianne has an initial endowment 10 kilos of beef and 32 litres of wine. Each has a utility function

$$U(B, W) = B\,W\,,$$

where B denotes the amount of beef consumed in kilos and W denotes the amount of wine consumed in litres. They can exchange beef and wine in a market, which operates according to the following rules. The market organizer announces prices P_B for beef and P_W for wine. Taking these prices as given, each calculates the value of his or her endowment, and regarding this as the income, decides how much beef and wine to consume, the value of the expenditure being constrained by the value of the endowment. Each announces truthfully the demand functions that result from this calculation. The market organizer then chooses the prices so as to ensure that both markets clear, that is, the amount of each good that the two together want to consume equals the total amount of that good available from their initial endowments.

(a) Write down the two consumers' budget constraints.

(b) Write down their demand functions using standard results. Is each of these functions homogeneous of degree zero in the prices P_B and P_W? Why, or why not?

(c) Show that for the beef market to clear, the market organizer must set $P_W/P_B = 1/2$.

(d) Does a different relation between P_B and P_W emerge from the requirement that the wine market should clear? Why, or why not?

(e) Is it possible to determine the separate values of P_B and P_W? Why, or why not?

(f) Now suppose a government confiscates both people's endowments of beef and wine, and redistributes them to achieve a Pareto efficient consumption bundle. We know that this requires equalization of John's and Marianne's marginal rates of substitution (MRS) between the two goods. Writing (B_J, W_J) for John's consumption quantities, find an expression for his MRS. Writing (B_M, W_M) for Marianne's consumption quantities, find an expression for her MRS. Equating the two, what can you infer about the locus of all Pareto efficient allocations?

(g) In a rough sketch of the box diagram showing the total quantities of beef and wine in this two-person economy on the two axes, show the endowment point, the locus of all Pareto efficient allocations, and the core, that is, the subset of these efficient allocations that can result from voluntary negotiations between the two persons.