This was graded by Brishti Guha, and the distribution was as follows:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>90–99</td>
<td>43</td>
</tr>
<tr>
<td>80–89</td>
<td>28</td>
</tr>
<tr>
<td>70–79</td>
<td>5</td>
</tr>
<tr>
<td>&lt;70</td>
<td>6</td>
</tr>
</tbody>
</table>

**Question 1: (50 points)**

COMMON ERRORS: (1) Forgetting about the boundary solution in part (a), and the second-order conditions in part (b). (2) Some carelessness in part (b), e.g. finding the quantities that minimize \( LRAC \) and \( SRTAC \), but forgetting to state the minimum values of these two entities. (3) Confusion between one firm’s supply curve and the industry supply curve in the long run. (4) Errors in the dead-weight loss calculations in part (e), including forgetting about the producers’ surplus.

(a) (4 points) Each consumer maximizes

\[
U(q,y) = y + 10q - 5q^2,
\]

subject to the budget constraint

\[
pq + y = M.
\]

Substituting out \( y \), the objective is

\[
F(q) = M - pq + 10q - 5q^2.
\]

Now

\[
F'(q) = 10 - p - 10q, \quad F''(q) = -10 < 0.
\]

So the FONC yields the global max, and

If \( p \geq 10 \), then \( F'(0) \leq 0 \) so \( q = 0 \) is optimum

If \( p < 10 \), then \( F'(0) > 0 \) and the optimum is \( q = 1 - p/10 \).

(3 points) The market demand function just the sum over 160 consumers:

\[
\text{If } p \geq 10, \text{ then } Q = 0
\]

\[
\text{If } p < 10, \text{ then } Q = 160 - 16p.
\]
(b) Other than the \( LRTC \) which is as stated, the other costs as functions of \( q \) are meaningful only for \( q > 0 \). The expressions are (5 points):

\[
\begin{align*}
LRTC(q) &= \begin{cases} 
0 & \text{if } q = 0 \\
4 + q^2 & \text{if } q > 0 
\end{cases} \\
LRAC(q) &= 4/q + q \\
SRTAC(q) &= 1 + q^2 \\
SRAAC(q) &= 1/q + q \\
MC(q) &= 2q
\end{align*}
\]

(2 points) \( LRAC'(q) = -4/q^2 + 1 \), \( LRAC''(q) = 8/q^3 > 0 \). So \( LRAC(q) \) is minimized at \( q = 2 \) and the minimum is 4.

(2 points) \( SRAAC'(q) = -1/q^2 + 1 \), \( SRAAC''(q) = 2/q^3 > 0 \). So \( SRAAC(q) \) is minimized at \( q = 1 \) and the minimum is 2.

(2 points) The firm’s short run supply curve coincides with its marginal cost curve so long as the price does not fall below the minimum \( SRAAC \). Thus \( p = 2q \) when \( p \geq 2 \). Or

\[
q = \begin{cases} 
0 & \text{if } p \leq 2 \\
p/2 & \text{if } p \geq 2 
\end{cases}
\]

Observe that if \( p = 2 \), the firm is indifferent between producing 0 and 1 (it merely loses its sunk cost, and only just recovers its avoidable cost if it produces 1). No points taken off this time for the finickiness of allowing for \( p = 2 \) in both cases. (4 points for figure below)

(c) (2 points) In the long run, with free entry and exit of fishing firms, the industry’s supply curve is a horizontal line at the level of the minimum \( LRAC \), that is, at \( p = 4 \). To be more pedantic, it consists of all points with \( p = 4 \) and \( q = 2, 4, 6, 8, 10, \ldots \). No points off for not being pedantic.

(d) (4 points) From the supply curve, \( p = 4 \). Then from the demand curve, \( Q = 16(10 - 4) = 96 \). Each firm produces \( q = 2 \) at the bottom point of its \( LRAC \), so there are 48 firms. Price equals the \( LRAC \) at this point, so each makes zero profit. The aggregate consumer surplus is the area to the left of the market demand curve, so it equals \( \frac{1}{2} (10 - 4) 96 = 288 \).
(e) (5 points) In the short run with 48 firms, the industry supply curve is

\[ Q = \begin{cases} 
0 & \text{if } p \leq 2 \\
24p & \text{if } p \geq 2
\end{cases} \]

Assuming for the moment that the 48 firms remain active, the price received by the firms is given as a function of the market quantity by

\[ p = Q / 24. \]

The price paid by the consumers is given as a function of \( Q \) by

\[ p = 10 - Q / 16. \]

With the tax, the equilibrium \( Q \) is defined by

\[ 10 - Q / 16 = 2.5 + Q / 24, \quad \text{or} \quad Q \left( \frac{1}{16} + \frac{1}{24} \right) = 7.5, \quad \text{or} \quad Q \frac{5}{48} = 7.5, \quad \text{or} \quad Q = 72. \]

(8 points for the rest of this part) Then the consumers pay the price \( p = 10 - 4.5 = 5.5 \). Firms receive the price \( 72 / 24 = 3 \). Since this exceeds the minimum \( SRAAC \), firms will not exit in the short run. Each of the 48 firms produces 1.5 units. So its revenue is 4.5, and its total cost is \( 4 + (1.5)^2 = 6.25 \). It makes a loss of 1.75, but that is less than its sunk cost namely 3. The total loss of the 48 firms is 84 Arials.

As the price paid by consumers rises from 4 to 5.5, and the quantity they buy shrinks from 96 to 72, the aggregate loss of consumer surplus is the trapezoid

\[ (5.5 - 4) \frac{96 + 72}{2} = 1.5 \times 84 = 126 \]

The government collects \( 2.5 \times 72 = 180 \) Arials. The dead-weight loss is \( 84 + 126 - 180 = 30 \) Arials. You could also calculate it as the area of the triangle

\[ \frac{1}{2} (5.5 - 3) (96 - 72). \]

(f) (2 points) Firms will want to exit in the long run. As some exit, the price rises and the remaining ones lose less, eventually breaking even again. In the new long run equilibrium,
the active firms must again be producing at their minimum LRAC, and this must equal the price they receive. So the price received by firms equals 4. Then that paid by consumers equals $4 + 2.5 = 6.5$. At this price, the quantity demanded is $16 (10 - 6.5) = 56$.

(7 points for the rest of this part) Since each firm produces 2, there must be 28 active firms. Each makes zero profit. The aggregate loss of consumer surplus (as compared to the old long run equilibrium) is

$$\frac{(6.5 - 4) \times (96 + 56)}{2} = 2.5 \times 76 = 190.$$ 

The government’s revenue is $2.5 \times 56 = 140$. The dead-weight loss is $190 - 140 = 50$. You could also calculate it as the area of the triangle

$$\frac{1}{2} (6.5 - 4) (96 - 56).$$

Extra information: (1) You were not asked to draw a figure for the dead-weight loss calculations, but you may find it useful (see top of next page). (2) The dead-weight loss is larger the longer the run, because more substitution gets made and more potentially valuable economic activity fails to take place.

**Question 2: (50 points)**

COMMON ERRORS: (1) Some thought that the core was the whole area inside the lens-shaped or football-shaped area bounded by the indifference curves through the endowment point. Actully it is only the part of the Pareto efficient points’ locus bounded by the two points where it intersects these indifference curves. (2) Not understanding the role of Walras’ law.

(a) (5 points) Writing $(B_J,W_J)$ for John’s consumption quantities, and $(B_M,W_M)$ for Marianne’s consumption quantities, the budget constraints are

$$P_B B_J + P_W W_J = 10 P_B + 8 P_W, \quad P_B B_M + P_W W_M = 10 P_B + 32 P_W.$$
(b) (8 points) Each utility being Cobb-Douglas with powers 1 and 1 for the two quantities, the budget shares are $1/(1 + 1) = 1/2$ for each good and for each person. Therefore the demand functions are: for John

$$B_J = \frac{10 P_B + 8 P_W}{2 P_B} = 5 + 4 \frac{P_W}{P_B}, \quad W_J = \frac{10 P_B + 8 P_W}{2 P_W} = 5 \frac{P_B}{P_W} + 4,$$

and for Marianne

$$B_M = \frac{10 P_B + 32 P_W}{2 P_B} = 5 + 16 \frac{P_W}{P_B}, \quad W_M = \frac{10 P_B + 32 P_W}{2 P_W} = 5 \frac{P_B}{P_W} + 16.$$

(2 points) Each is homogeneous of degree zero in $(P_B, P_W)$, because any common scale factor applied to both prices cancels out of the right hand sides. Another way to state homogeneity of degree zero is that the demands are functions only of the relative price or price ratio $P_B/P_W$.

(c) (5 points) For the beef market to clear, we need

$$5 + 4 \frac{P_W}{P_B} + 5 + 16 \frac{P_W}{P_B} = 20, \quad \text{or} \quad 20 \frac{P_W}{P_B} = 10,$$

or $P_W/P_B = 1/2$.

(d) (3 points) For the wine market to clear, we need

$$5 \frac{P_B}{P_W} + 4 + 5 \frac{P_B}{P_W} + 16 = 40, \quad \text{or} \quad 10 \frac{P_B}{P_W} = 20,$$

or $P_B/P_W = 2$.

(5 points) which is the same as $P_W/P_B = 1/2$, the condition for the beef market to clear. This is an implication of Walras’ Law. Since the values of the excess demands in the two markets sum identically to zero, when the excess demand quantity in one market equals zero, so must that in the other market.

(e) (4 points) It is not possible to determine the separate values of $P_B$ and $P_W$. All quantity choices depend only on the price ratio; therefore the equilibrium conditions can only solve for that ratio.

(f) (6 points) John’s MRS is

$$-\frac{dW_J}{dB_J} \bigg|_{U(B_J,W_J)=\text{constant}} = \frac{\partial U/\partial B_J}{\partial U/\partial W_J} = \frac{W_J}{B_J}.$$

Similarly, Marianne’s MRS is $W_M/B_M$. Pareto efficient allocations must therefore maintain equal ratios of wine/beef between the two, and then the common value of the two ratios must also equal the ratio of wine/beef that is available in total:

$$\frac{W_J}{B_J} = \frac{W_M}{B_M} \implies \text{each is } \frac{W_J + W_M}{B_J + B_M} = \frac{40}{20} = 2.$$
(g) (12 points) The figure is shown below. Notation: \(O_J, O_M\) are the origins, \(E\) the endowment point, \(IC_J, IC_M\) the indifference curves through \(E\). The diagonal joining \(O_J\) and \(O_M\) is the locus of Pareto efficient allocations (because in this example, efficiency requires the ratios of the two goods to be equal for the two people). The thicker portion \(JM\) is the core.

We can actually calculate the coordinates of the points \(J\) and \(M\). Since \(W_J\) and \(M_J\) are on the pareto efficient locus, \(W_J/B_J = 2\). Since they are on John’s indifference curve through the initial point, \(W_J B_J = 8 \times 10 = 80\). Therefore \(2 (B_J)^2 = 80\), or \(B_J = \sqrt{40} = 2 \sqrt{10}\), and then \(W_J = 4 \sqrt{10}\). Similarly, \(W_M = 2 B_M\) and \(W_M B_M = 32 \times 10 = 320\), so \(2 (B_M)^2 = 320\) or \(B_M = \sqrt{160} = 4 \sqrt{10}\), and then \(W_M = 8 \sqrt{10}\). No points taken off for not calculating these numbers. In fact we would have given a few bonus points to anyone who did calculate them. But some of you merely drew a general “schematic diagram” like that of the lecture handout. You have to be more precise in applying it to this problem - identify the endowment point, show the exact Pareto efficient locus, and so on.

Additional information: Part of \(IC_M\) is shown extending outside the box. This was an accident of combining graphs from two different software packages, but actually there is nothing economically wrong with this. The box represents the total quantities of goods in the economy. One person can perfectly well have preferences over quantities of goods greater than that. It is the job of the overall mechanism of resource allocation in the economy, whether it be a market equilibrium, or private negotiation between the two parties, or a government, to ensure that the resulting allocation is feasible, and in particular, that it does not give to one person (or even all of them taken together) more than the total that is available.