ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 6 – Due November 20 in class

Question 1: (20 points)

Toogoodtobetrue, Inc. has a monopoly over a good that is costless to produce. The demand comes from two types of consumers. There are 800 consumers of type 1 and 200 consumers of type 2. Each consumer of type 1 has the demand function

$$q_1 = 1 - \ln(p)\,,$$

and each consumer of type 2 has the demand function

$$q_2 = 2 - \ln(p)$$

Ignore the problem that for sufficiently high prices these quantities become negative; the actual prices will not go into this range.

(a) If the firm has to set a uniform price p for all consumers, what is its profit-maximizing choice?

(b) If the firm can set separate prices, p_1 for type-1 consumers and p_2 for type-2 consumers, what are its profit-maximizing choices?

(c) When the firm can charge separate prices, which group pays a higher price? Why?

Question 2: (40 points)

(Note: Despite the apparent symmetry between the two firms in this problem, you should not start with an assumption that the prices or quantities are going to be equal. Where that is true, you should find it as a part of your solution. However, you should ignore issues of second-order conditions, non-negativity of prices and quantities etc. – they are all OK here.)

There are two goods, whose quantities are denoted by x_1, x_2 and prices by p_1, p_2. The demand functions are

$$x_1 = 2 - p_1 + k\,p_2, \qquad x_2 = 2 + k\,p_1 - p_2\,.$$

Here k is an algebraic constant (parameter) whose value is somewhere in the range $-1 < k < 1$.

Each good is produced by one firm. The marginal cost of production of each good is constant and equal to 1.

(a) For what range of possible values of k are the two goods substitutes? When are they complements?

(b) Obtain expressions for the profits Π_1 and Π_2 of the two firms in terms of their prices (and the constant k).

(c) First suppose that the firms are price-setting duopolists, each trying to maximize its own profit. Find their best response functions. The Bertrand-Nash equilibrium of the duopoly is defined by the simultaneous solution of these best response functions for the two prices. Find the prices and profits in the Bertrand-Nash equilibrium. (The solutions will have k on the right hand side).

(d) Next suppose the two firms collude and choose their prices to maximize their joint profit $\Pi_1 + \Pi_2$, and then share this out equally between the two firms. Find the resulting prices and profits, again as expressions in terms of k.

(e) Compare the solutions in (c) and (d). For what range of values of k are the prices in (d) higher than the prices in (c)? For what range of values of k are the profits in (d) higher than the profits in (c)?

(f) Find the economic intuition for your answers in (e).

Question 3: (40 points)

Sandy Beach stretches in a straight line for 1 mile. Sunbathers are spread uniformly along this mile. Two ice cream vendors, Jen and Berry, have kiosks located at the two end-points of the beach. They sell identical kinds and sizes of cones. The cost of each is \$2. Denote the prices by P_j and P_b respectively.

Each sunbather buys one cone. To get a cone, they have to walk to one of the kiosks. Each buyer regards a walk of x miles and back (where x is a fraction in this problem) exactly like spending \$ x. So a customer located at distance x from Jen's kiosk has to "pay" a total of $(P_j + x)$ to buy from Jen's kiosk and $(P_b + 1 - x)$ to buy from Berry's kiosk. Each consumer chooses the kiosk that minimizes the total he or she "pays".

Jen wants to maximize her profit, and Berry wants to maximize his profit. Note that the customers' walking costs do not become the sellers' revenues; the sellers get only the prices they charge.

(a) Express the quantities demanded from Jen and Berry, measured in units of "miles of customers", each as a function of the two prices P_j and P_b.

(b) Express Jen's and Berry's profits, Π_j and Π_b, as functions of the two prices P_j and P_b.

(c) Find the Bertrand-Nash equilibrium of this pricing duopoly game.

(d) Now suppose Jen and Berry get together and choose a common price P, in an attempt to maximize their joint profit. However, they realize that \$4 is the most in total (price plus walking cost) that each customer is willing to "pay" for a cone. (Observe that the prices P_j and P_b you found in the equilibrium of section (c) automatically satisfied this condition. That is why it was not introduced there.) What is the highest P for which even the customer farthest from the kiosks (the one at exactly the mid-point of the mile) is willing to buy? Do Jen and Berry want to push their collusive common price P this high?

(e) If the two sellers raise their price even higher, they will lose some customers in the middle part of the mile, although they will get higher revenues from the customers who are closer to each and continue to buy. Would such a price increase be in their interest?

(f) What is the optimal collusive profit for each?