ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 7 – Due December 4 in class

Question 1: (40 points)

Note: Do all the calculations for this problem in units of \$1 million (megabucks). Be especially careful with your algebra and arithmetic in this question.

Your initial wealth is \$1 million. You can invest a proportion x of it in stocks, a proportion y in bonds, and the rest in cash. Cash always yields zero return, therefore that part of your wealth stays at $(1 - x - y)$ in all scenarios. There are five scenarios:

(1) The Ho-Hum Scenario (probability 40%): The values of stocks and bonds do not change at all.

(2) The Goldilocks Economy (probability 20%): Everything is exactly right; the economy prospers and inflation is low. The value of stocks doubles, and that of bonds goes up by 50%.

(3) Stagflation (probability 20%): The economy stagnates and interest rates go up. The value of stocks halves, and that of bonds goes down by 25%.

(4) Inflation (probability 10%): The economy booms but interest rates rise sharply. The value of stocks doubles, but that of bonds goes down by 25%.

(5) Deflation (probability 10%): The economy does badly and interest rates are low. The value of stocks halves, but that of bonds goes up by 50%.

Ignore the dividends on the stocks and the interest on the bonds; these are negligible compared to the changes in the values of the assets stated above.

(a) Write down expressions for your final wealth, denoted by respectively W_1, W_2, ... W_5, in each of these five scenarios, in each case as a function of x and y.

(b) Suppose your von Neumann-Morgenstern utility function is

$$U(W) = W - \tfrac{1}{4}\, W^2\,.$$

Write down the expression for your expected utility, as a function of W_1, W_2, ... W_5.

(c) Find the values of x and y that maximize your expected utility. (Do not worry about second-order conditions or boundary solutions in this part.)

(d) Do not derive any calculus second-order conditions, but say in a couple of sentences why expected utility is here a concave function of (x, y) ensuring that the SOCs are satisfied.

Question 2: (30 points)

You have initial wealth W_0 dollars. With probability p you will suffer a disaster that will wipe out this wealth completely; otherwise it will stay intact. You can insure against this loss. Denote by q the premium per dollar of insurance. This means that if you buy X dollars of insurance coverage, you have to pay $q\,X$ dollars right now, and will get X dollars from the insurance company if you suffer the disaster and nothing if you do not.

(a) Insurance is supplied by risk-neutral companies in a competitive insurance market. If a claim for X dollars arises, the company must incur an administrative cost of $c\,X$ dollars to investigate and process it. Find the expected profit of an insurance company on a contract for X dollars of insurance coverage. If competition ensures zero expected profit on each such contract, what relation must link q, p, and c?

(b) Suppose you have a von Neumann-Morgenstern utility function with a constant coefficient of relative risk aversion r. Find the expression for your expected utility when you buy X dollars of insurance coverage.

(c) By maximizing this expected utility with respect to X, find a formula for the fraction X/W_0 of your loss that you will choose to cover, as a function of q, p, and r.

(d) Numerically evaluate this, taking $p = 0.1$, two cases of c, namely $c = 0.1$ and $c = 0.2$, and three cases of r, namely $r = 0.25$, $r = 1$, and $r = 10$ (six calculations in all). In each case, the price of insurance q is to be set at its competitive equilibrium level.

Question 3: (30 points)

Consider two college roommates, Doc the premed, and Geek the computer science major. They recognize that a doctor will have a steady income, whereas a computer scientist's income will depend on the fate of his dotcom. They have both just completed ECO 305, so they propose to trade Arrow-Debreu securities to achieve an optimal allocation of the risk. Assume that each acts as a price-taker in the markets for Arrow-Debreu securities.

They have calculated that Doc's wealth will be \$10 million no matter what. Geek's wealth will be \$50 million if his dotcom flourishes, and 0 if it collapses to a mere $\cdot$. The probability that the dotcom flourishes happens is 40%, and the probability that it collapses is 60%.

Each has a logarithmic von Neumann-Morgenstern utility function of wealth.

(a) What are the "scenarios" ?

(b) An Arrow-Debreu security for each scenario is defined as a contract that will pay 1 megabuck in that scenario and nothing otherwise. What are the two students' endowments of Arrow-Debreu securities? Writing P_i for the price (in today's trading) of the Arrow-Debreu security for the scenario labeled i, write down the values of the two students' endowments of these securities.

(c) Writing W_i^j for the final wealth of the student labeled j (where j stands for Doc or Geek) in the scenario labeled i (that is, he ends up with W_i^j after the scenario is realized and the claims implied by his trades in Arrow-Debreu securities have been settled), what are the two students' budget constraints for today's trades in Arrow-Debreu securities?

(d) Write down expressions for the two students' expected utilities. Write down their demand functions for these securities.

(e) Find the equilibrium relative price of the securities, and the magnitudes of the two students' final wealths in the scenarios.

(Note: This question assumes that the two students act as price-takers in the markets for the two Arrow-Debreu securities. Since there are only two of them, you will doubt the validity of this assumption. Just accept it for now; I will comment on it in the answer key.)