ECO 305 – Fall 2003
Microeconomic Theory – A Mathematical Approach
Problem Set 7 – Answer Key

Brishti Guha graded this. But she has left early for the Christmas break, so if after studying the answer key you have any remaining questions, please bring them to me.

Many people took one of their freebies on this one, but those who turned in their answers generally did very well. We were giving a couple of bonus points (see below) so some even got more than 100.

The distribution was as follows:

Range	Number
≥ 100	25
90–99	39
80–89	8
< 70	3

With these scores there are very few common errors, but the few are listed below.

Question 1: (40 points)

ERROR: A few people thought that x and y had to add up to 1, and "manipulated" their numbers to make it so.

(a) (10 points) Expressions for the final wealth in the five scenarios

$$\begin{aligned}
W_1 &= (1-x-y)+x+y = 1\\
W_2 &= (1-x-y)+2\,x+1.5\,y = 1+x+0.5\,y\\
W_3 &= (1-x-y)+0.5\,x+0.75\,y = 1-0.5\,x-0.25\,y\\
W_4 &= (1-x-y)+2\,x+0.75\,y = 1+x-0.25\,y\\
W_5 &= (1-x-y)+0.5\,x+1.5\,y = 1-0.5\,x+0.5\,y
\end{aligned}$$

(b) (5 points) Expected utility

$$\begin{aligned}
EU &= 0.4\,[W_1 - \tfrac{1}{4}\,(W_1)^2] + 0.2\,[W_2 - \tfrac{1}{4}\,(W_2)^2] + 0.2\,[W_3 - \tfrac{1}{4}\,(W_3)^2]\\
&\quad +0.2\,[W_4 - \tfrac{1}{4}\,(W_4)^2] + 0.2\,[W_5 - \tfrac{1}{4}\,(W_5)^2]
\end{aligned}$$

(c) (22 points - 8 for each derivative and 6 for correct joint solution of the two FONCs.)

NOTE: I keep the expression for EU as in (b) and use the chain rule to differentiate with respect to x and y. This is easier, and less liable to error, than expanding out EU explicitly in terms of x and y and then differentiating.

The FONCS for EU-maximization:

$$\begin{aligned}
\frac{\partial EU}{\partial x} &= 0.2\,[1-\tfrac{1}{2}\,W_2]\,\frac{\partial W_2}{\partial x} + 0.2\,[1-\tfrac{1}{2}\,W_3]\,\frac{\partial W_3}{\partial x}\\
&\quad +0.1\,[1-\tfrac{1}{2}\,W_4]\,\frac{\partial W_4}{\partial x} + 0.1\,[1-\tfrac{1}{2}\,W_5]\,\frac{\partial W_5}{\partial x}\\
&= 0.2\,[1-\tfrac{1}{2}\,(1+x+0.5\,y)]\,(1) + 0.2\,[1-\tfrac{1}{2}\,(1-0.5\,x-0.25\,y)]\,(-0.5)\\
&\quad +0.1\,[1-\tfrac{1}{2}\,(1+x-0.25\,y)]\,(1) + 0.1\,[1-\tfrac{1}{2}\,(1-0.5\,x+0.5\,y)]\,(-0.5)\\
&= 0.075 - 0.1875\,x - 0.0375\,y = 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial EU}{\partial x} &= 0.2\ [1 - \tfrac{1}{2}\, W_2]\ \frac{\partial W_2}{\partial y} + 0.2\ [1 - \tfrac{1}{2}\, W_3]\ \frac{\partial W_3}{\partial y} \\
&\quad +0.1\ [1 - \tfrac{1}{2}\, W_4]\ \frac{\partial W_4}{\partial y} + 0.1\ [1 - \tfrac{1}{2}\, W_5]\ \frac{\partial W_5}{\partial y} \\
&= 0.2\ [1 - \tfrac{1}{2}\,(1 + x + 0.5\ y)]\ (0.5) + 0.2\ [1 - \tfrac{1}{2}\,(1 - 0.5\ x - 0.25\ y)]\ (-0.25) \\
&\quad +0.1\ [1 - \tfrac{1}{2}\,(1 + x - 0.25\ y)]\ (-0.25) + 0.1\ [1 - \tfrac{1}{2}\,(1 - 0.5\ x + 0.5\ y)]\ (0.5) \\
&= 0.0375 - 0.0375\ x - 0.046875\ y = 0
\end{aligned}
$$

You can solve these as they are; they also simplify to

$$5\ x + y = 2, \qquad 4\ x + 5\ y = 4$$

The solutions are

$$x = 2/7 = 0.286, \qquad y = 4/7 = 0.571$$

(Then the fraction held in cash is $1 - x - y = 1/7 = 0.143$.)

(d) (3 points) The wealth in each scenario is a linear function of x and y, the von Neumann-Morgenstern utility in each scenario is a concave function of the wealth in that scenario, and expected utility is a positive linear combination of the von Neumann-Morgenstern utilities. Therefore expected utility is a concave function of (x, y).

Question 2: (30 points)

ERROR: The question states that the accident wipes out "this wealth", that is W_0. A few people thought that loss was only $W_0 - qX$. We gave partial credit for following through this correctly.

(a) (5 points) The expected profit of the insurance company is $q\,X - p\,X - p\,c\,X = [q - (1+c)p]\,X$. (You might have thought that the administration cost was cX. But remember, this cost is incurred only if a claim is made, that is, with probability p. Therefore the expected cost is pcX.) With several perfectly competitive insurance companies, in equilibrium the expected profit must be zero, therefore $q = (1+c)\,p$.

(b) (5 points) Call the scenario in which you suffer the loss scenario 1, and the one where your wealth stays intact, scenario 2. If you take out coverage X, your expected utility is

$$EU = \frac{1}{1-r}\ \left[\,p\,\{(1-q)X\}^{1-r} + (1-p)\,\{W_0 - qX\}^{1-r}\,\right]$$

when $r \neq 1$ (and the log case if $r = 1$).

(c) (10 points) To choose X to maximize this, the FONC is

$$\frac{dEU}{dX} = p\,\{(1-q)X\}^{-r}\,(1-q) + (1-p)\,\{W_0 - qX\}^{-r}\,(-q) = 0$$

which simplifies to give the required expression for the fraction of your loss that is covered:

$$\frac{X}{W_0} = \left\{ q + (1-q)\ \left[\frac{q/(1-q)}{p/(1-p)}\right]^{1/r} \right\}^{-1}$$

Second-order conditions are OK because the wealth in each scenario is a linear function of X, the von Neumann-Morgenstern utility function in each scenario is a concave function of the wealth in that scneario, and expected utility is a postive linear combination of the vN-M utilities in the various scenarios.

The derivative dEU/dx has the same functional form for all cases of r, so you actually don't need to do the log case separately. We gave 2 bonus points to those who did.

(d) (10 points) When $p = 0.10$, for two values of the administrative cost factor $c = 0.1$ and 0.2, we have $q = 0.11$ and 0.12 respectively. Then, for the three values of the risk aversion coefficient r given, we have the following table for the resulting values of the coverage ratio X/W_0:

Cost factor c	Relative risk aversion r 0.25	1.0	10.0
0.1	0.6791	0.9090	0.9906
0.2	0.4725	0.8333	0.9821

Question 3: (30 points)

ERRORS: (1) A few people got confused about the budget constraints. (2) Some thought that Doc would not demand any Scenario-2 ADS's because Geek has no wealth in that scenario. Actually Doc has a demand for these, but in equilibrium it is less than hiw own endowment, so he becomes a net seller. (3) Some started treating prices as probabilities and said they must sum to 1. ADS prices summing to 1 is a separate issue, to do with whether it is possible to transfer wealth 1-for-1 from before the event to after the resolution of uncertainty.

(a) (2 points) Scenario 1: Dotcom prospers; total wealth = 60 (megabucks)
Scenario 2: Dotcom is toast; total wealth = 10

(b) (4 points) Endowments: Doc - (10,10); Geek (50,0).
Values Doc: $10\ P_1 + 10\ P_2$; Geek: $50\ P_1$

(c) (4 points) Budget constraints:

$$\begin{aligned}
\text{Doc:} \quad & P_1\ W_1^D + P_2\ W_2^D = 10\ P_1 + 10\ P_2 \\
\text{Geek:} \quad & P_1\ W_1^G + P_2\ W_2^G = 50\ P_1
\end{aligned}$$

(d) (4 points) Expected utilities:

$$\begin{aligned}
\text{Doc:} \quad & 0.4\ \ln(W_1^D) + 0.6\ \ln(W_2^D) \\
\text{Geek:} \quad & 0.4\ \ln(W_1^G) + 0.6\ \ln(W_2^G)
\end{aligned}$$

(8 points) Demand functions (standard Cobb-Douglas formulas)

$$\begin{aligned}
\text{Doc:} \quad & W_1^D = 0.4 \frac{10\ P_1 + 10\ P_2}{P_1} = 4 + 4\ \frac{P_2}{P_1} \\
& W_2^D = 0.6\ \frac{10\ P_1 + 10\ P_2}{P_2} = 6\ \frac{P_1}{P_2} + 6 \\
\text{Geek:} \quad & W_1^G = 0.4\ \frac{50\ P_1}{P_1} = 20, \quad W_2^G = 0.6\ \frac{50\ P_1}{P_2} = 30\ \frac{P_1}{P_2}
\end{aligned}$$

(e) (4 points) For equilibrium in the market for scenario-1 Arrow-Debreu securities:

$$4 + 4\ \frac{P_2}{P_1} + 20 = 60, \qquad \text{so} \qquad \frac{P_2}{P_1} = 9$$

(We don't need to look at equilibrium in the other market, and don't need the absolute prices of the two securities, for the usual reasons – Walras' Law and homogeneity.)

(4 points) Then the demand functions give the final wealth amounts:

$$\begin{aligned}
\text{Doc:} \quad & W_1^D = 40, \quad W_2^D = 6\tfrac{2}{3} \\
\text{Geek:} \quad & W_1^G = 20, \quad W_2^G = 3\tfrac{1}{3}
\end{aligned}$$

Additional information: Observe that Geek has twice as much expected wealth as Doc: $0.4 \times 50 + 0.6 \times 0 = 20$ versus $0.4 \times 10 + 0.6 \times 10 = 10$. But Doc ends up with twice as much as Geek in each scenario, because he has more wealth in scenario 2, which is much more valuable.

Comment on the price-taking assumption: When there are only two people trading with each other, you expect them to be aware of this and engage in bargaining, not passive price-taking. So something like a core, or a more refined theory of bargaining leading to a unique outcome, would be better than the supply-demand-equilibrium model. You could suppose that each of these two stands for hundreds of similar people with identical demands. That would work fine in the exchange problems we studied before, for example John and Marianne exchanging beef and wine in Problem Set 5. But here an additional issue arises. For there to be just two scenarios even when there are hundreds of Geeks, the nature of uncertainty must be such that all the dotcoms either succeed or fail together. That is, the uncertainty about the success of all the Geeks' dotcomes must be perfectly correlated. Otherwise, with n Geeks, one has to recognize 2^n different scenarios, corresponding to the whole list of successes and failures. Of these, there are only $(n + 1)$ different levels of *a*ggregate risk, corresponding to the total number of dotcomes that succeed or fail, and the probability distribution of aggregate risk is binomial if the risks are independent. Within each level of aggregate risk, there will be some lucky and some unlucky Geeks; that is *i*ndividual risk. See the Cora-Ira example in the long handout "Financial Markets" to see how such risks are priced in the market.