The distribution was as follows:

<table>
<thead>
<tr>
<th>Range</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-99</td>
<td>1</td>
</tr>
<tr>
<td>80-89</td>
<td>4</td>
</tr>
<tr>
<td>70-79</td>
<td>6</td>
</tr>
<tr>
<td>60-69</td>
<td>14</td>
</tr>
<tr>
<td>50-59</td>
<td>25</td>
</tr>
<tr>
<td>40-49</td>
<td>16</td>
</tr>
<tr>
<td>30-39</td>
<td>12</td>
</tr>
</tbody>
</table>

And here are some question-by-question statistics

<table>
<thead>
<tr>
<th></th>
<th>Q. 1</th>
<th>Q. 2</th>
<th>Q. 3</th>
<th>Q. 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>16</td>
<td>18.6</td>
<td>14.4</td>
<td>5.6</td>
<td>54.5</td>
</tr>
<tr>
<td>Median</td>
<td>15</td>
<td>19</td>
<td>15</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td>Max</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>90</td>
</tr>
<tr>
<td>Min</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>32</td>
</tr>
</tbody>
</table>

To put it mildly, the performance was disappointing. To put it bluntly, it was awful. However, there are some redeeming aspects: [1] The scoring is on the full 0-100 scale, not a humanities-style “everyone gets a smiley” 60-100 scale. [2] There is strength in numbers; if everyone does badly, then your relative performance is less bad. [3] Last year the corresponding numbers were not too different: Mean 57.3, Median 58.5, Max 85, Min 26. Whether the students responded to this shock by working harder or for some other reason, all ended well; no one failed the course.

QUESTION 1: MULTIPLE CHOICE.

Much of this was ECO 100 material; disappointingly the median student got only 5 of the 8 correct.

1. d.
2. c.
3. b.
4. d
5. e. (When the price of any good – normal or inferior – rises, the substitution effect causes its quantity demanded to go down.)
6. d. (Gradgrind Tech offers the highest expected income; you are risk neutral.)
7. c.
8. a. (This is merely the formula for the MRTS in terms of the marginal products, and is therefore true at all points on all isoquants, not just for the cost-minimizing choices. So it is not any kind of condition for cost-minimization.)

**QUESTION 2:**

This was basically ECO 100 material, and you should also know it from P-R pp. 300-304 and class handout of Oct. 4 pp. 1-2. The precept of week 3 also did some of this.

This question was generally fairly well done. But two points should be made: [1] Some of you showed the wrong triangle as the dead-weight loss, namely a triangle to the left of the demand curve, with quantity above 250 and price above 1.2. In this example that triangle happens to have the same area as the correct triangle EFS because the demand curve is a straight line. So you got the right numerical answer but the concept was wrong; 3 points were taken off for this. [2] I thought I was making the numbers really easy for numerical calculation, but many of you had real trouble with calculating 0.8 times 350, for example. Teaching of mental arithmetic in schools must have gone really downhill. Martin was generous in grading arithmetic mistakes; I would have been harsher. But I think I am going to give up and allow calculators in future exams. (The downside is that allowing calculators may give an unfair advantage to people who have highly sophisticated calculators.)

8 points for the figure

![Diagram showing price and quantity with points A, E, F, B, S, and Q]

2 points: Originally, \( P = 2, \ Q = 500 – 2 \times 125 = 250 \)
2 points: After subsidy, \( P = 1.2, \ Q = 500 – 1.2 \times 125 = 500 – 150 = 350 \)
3 points: Revenue cost of subsidy = \( 0.8 \times 350 = 280 \)  (= rectangle ABSF in figure)
5 points: Consumer surplus gain = trapezoidal area ABSE in figure

\[
\text{= } \frac{1}{2} \times (AE + BS) \times AB = \frac{1}{2} \times (250 + 350) \times 0.8 = 300 \times 0.8 = 240
\]

5 points: Deadweight loss = Revenue cost – Consumer surplus gain = 280 – 240 = 40

Or directly, DWL = triangle area EFS in figure

\[
\text{= } \frac{1}{2} \times EF \times FS = \frac{1}{2} \times (350-250) \times 0.8 = \frac{1}{2} \times 100 \times 0.8 = 40
\]
QUESTION 3:

The performance on this was generally mediocre. Too many people didn’t know the basic concepts such as what is expected utility, what does it mean for a utility function to represent preferences etc. For each part I give an indication of where you should have known the material from.

(a) (5 points) P-R pp. 159-60: Expected utility is the sum of utilities associated with all possible outcomes, weighted by the probability that each outcome will occur. OR (Class handout Oct. 6 p. 3: If a consumer can have different outcomes (of wealth, income, consumption etc. depending on the specific context) x₁, x₂, … xₙ, with respective probabilities p₁, p₂, … pₙ, and his utility function over outcomes is U, then his expected utility is

$$EU = p₁ U(x₁) + p₂ U(x₂) + … pₙ U(xₙ)$$

A perfect answer would give both the statement and the formula, but we give full credit for a full correct version of either one. Actually most answers were very poor.

(b) (5 points) Risk-aversion corresponds to a concave utility function (MU = U’(x) decreasing, or U’’(x) < 0). An example: U(x) = ln(x).

(c) (5 points) P-R p. 162: Risk premium is the maximum amount of money the consumer is willing to pay to avoid the risk. Formal statement paralleling the answer to (a) above: Let E[x] = p₁ x₁ + p₂ x₂ + … + pₙ xₙ denote the average outcome in money or quantity terms. Then the risk premium R is the sure reduction in money or quantity below this level that would make the consumer indifferent between the reduced sure amount and the risky prospect, that is, the solution to the equation

$$U( E[x] – R ) = p₁ U(x₁) + p₂ U(x₂) + … pₙ U(xₙ)$$

You can also draw figures like that on P-R p. 162 or Class Handout Oct. 6 p. 4 when answering parts (b) and (c).

(d) (2 points) (This was discussed in precepts week 4, and Martin gave a great example of various temperature scales as the “affine” transformations that were permitted while preserving the ability of the utility function to represent the same preferences.)

$$V(x) = 3 + 7 \ln(x)$$ represents the same preferences as $$U(x) = \ln(x)$$. (Some of you said that $$V(x) = \sqrt{x} = x^{\frac{1}{2}}$$ would represent the same preferences as $$U(x) = \ln(x)$$. That is not true; the two correspond to different degrees of risk aversion. Using the coefficient of relative risk aversion concept from the class handout of Oct. 6 p. 5, you see that the square root function has $$\rho = \frac{1}{2}$$ and the log function has $$\rho = 1$$.

(e) (8 points) Criticism: Any section in Pindyck-Rubinfeld pp. 179-82 on behavioral economics, or the Allais paradox (class handout Oct. 11 p. 2) or Ellsberg paradox (precept week 4). Alternative: Prospect theory or Regret theory (class handout Oct. 11 p. 3). Your grade will depend on how complete, correct, and clear your answer is. This is Martin’s judgment call.
QUESTION 4:

This was the worst question, surprisingly so since you had done so many Cobb-Douglas maximization problems. Many of you recognized and said “Cobb-Douglas” (sometimes misspelling Douglas as Douglass), but were unable to do anything beyond that. “Cobb-Douglas” is not some kind of magical incantation or spell from Harry Potter (“Expecto Solutio!”). You can get the formulas for the optimal K and L using that, but must then use these formulas into your further work, in this case solving for M in terms of G or vice versa.

Some of you tried to do the full Lagrange solution, and made errors when taking derivatives etc.

Several of you had asked me about exactly this calculation in office hours and the evening study hall; Martin says the same kind of questions also came up in all of his precepts on the day before the exam. So I would have hoped that more people would ace this. And it can be done; three people got the full 25 points on this question, and two others got above 20. But then the scores drop suddenly to 12 and all the way to 1. And this despite quite generous grading by Martin.

See P-R pp. 258-60, class handout of October 20 p. 2.

The Cobb-Douglas formula gives the optimal allocation of your time:

\[
\frac{rK}{M} = \frac{0.1}{0.1 + 0.4} = 0.2, \quad \frac{wL}{M} = \frac{0.4}{0.1 + 0.4} = 0.8
\]

Therefore

\[
K = \frac{0.2 M}{r}, \quad L = \frac{0.8 M}{w}
\]

(10 points for this; 2 taken off if Cobb-Douglas is not cited correctly.)

Then

\[
G = A \left( \frac{0.2 M}{r} \right)^{0.1} \left( \frac{0.8 M}{w} \right)^{0.4} = A \left( 0.2 \right)^{0.1} \left( 0.8 \right)^{0.4} r^{-0.1} w^{-0.4} M^{0.5}
\]

Or

\[
M = 0.2^{-0.2} 0.8^{-0.8} A^{-2} r^{0.2} w^{0.8} G^2
\]

(8 points for this calculation)

Then marginal cost = \( \frac{dM}{dG} = 2 * 0.2^{-0.2} 0.8^{-0.8} A^{-2} r^{0.2} w^{0.8} G \)

Average cost = \( \frac{M}{G} = 0.2^{-0.2} 0.8^{-0.8} A^{-2} r^{0.2} w^{0.8} G \)

So MC = 2 AC. (7 points for this part)